## Structured Computation and Representation in Deep Reinforcement Learning

Jessica B. Hamrick



ICLR Workshop on Deep Reinforcement Learning Meets Structured Prediction

> New Orleans, LA, USA May 6, 2019

















The arrangement of and *relations* between the *parts* or elements of something complex.

(Oxford English Dictionary)

The arrangement of and *relations* between the *parts* or elements of something complex.

(Oxford English Dictionary)

In humans: Evolution → Structure

The arrangement of and *relations* between the *parts* or elements of something complex.

(Oxford English Dictionary)

In humans: Evolution → Structure

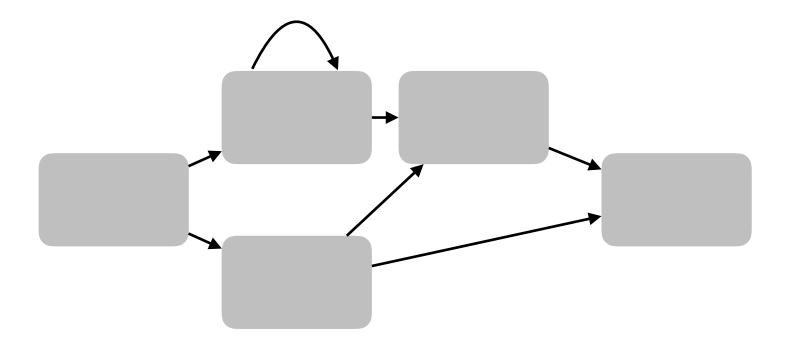
In ML: Structure → Inductive bias / prior / regularizer

## Two Types of Structure



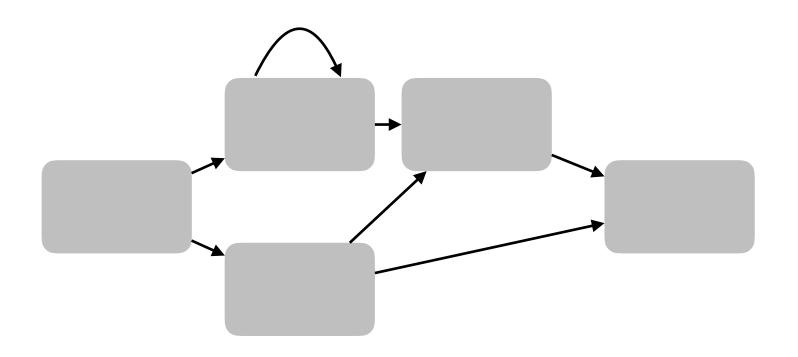
## Two Types of Structure

1. Structured Computation: the way that individual computations or functions are composed into more complex structures.

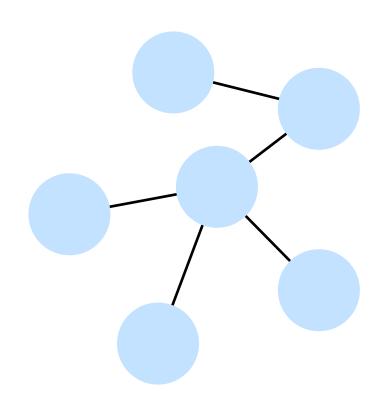


## Two Types of Structure

1. Structured Computation: the way that individual computations or functions are composed into more complex structures.



2. Structured Representation: the format of the data that computations are performed over, e.g. sets, graphs, programs, etc.





Structured computation and structured representation go hand in hand!

Structured computation and structured representation go hand in hand!

Often, a structured representation entails a particular structure in the computation, and vice versa.



Structured computation and structured representation go hand in hand!

Often, a structured representation entails a particular structure in the computation, and vice versa.

But it can still be useful to think separately about:

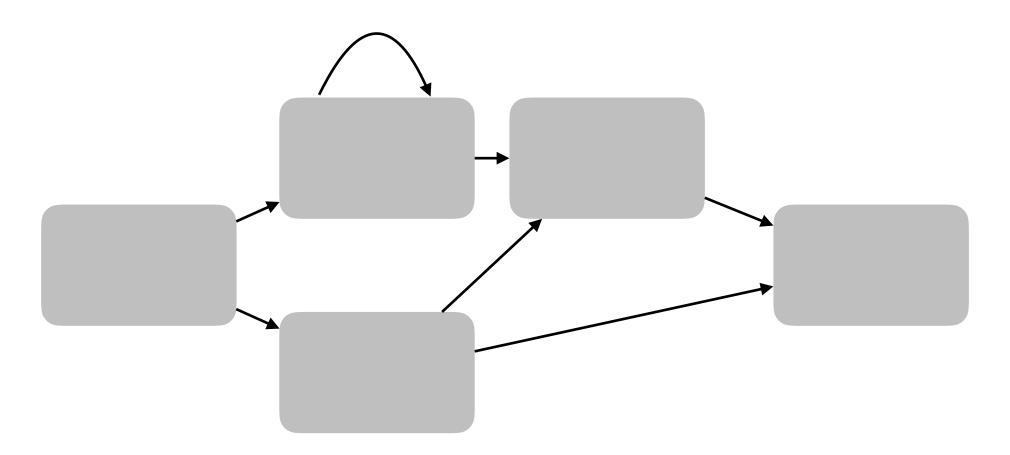
- (1) how computations are arranged and composed, and
  - (2) the specific form of the representations.

Structured computation and structured representation go hand in hand!

Often, a structured representation entails a particular structure in the computation, and vice versa.

But it can still be useful to think separately about:

- (1) how computations are arranged and composed, and
  - (2) the specific form of the representations.

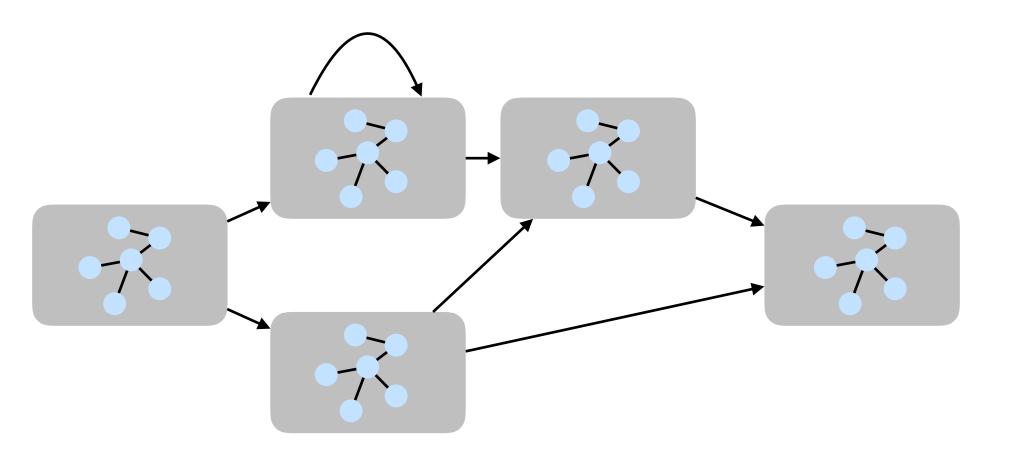


Structured computation and structured representation go hand in hand!

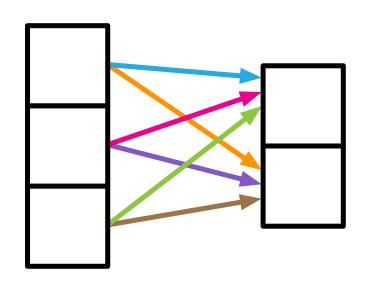
Often, a structured representation entails a particular structure in the computation, and vice versa.

But it can still be useful to think separately about:

- (1) how computations are arranged and composed, and
  - (2) the specific form of the representations.

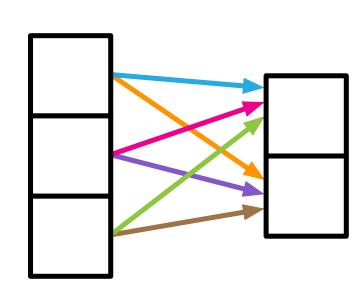




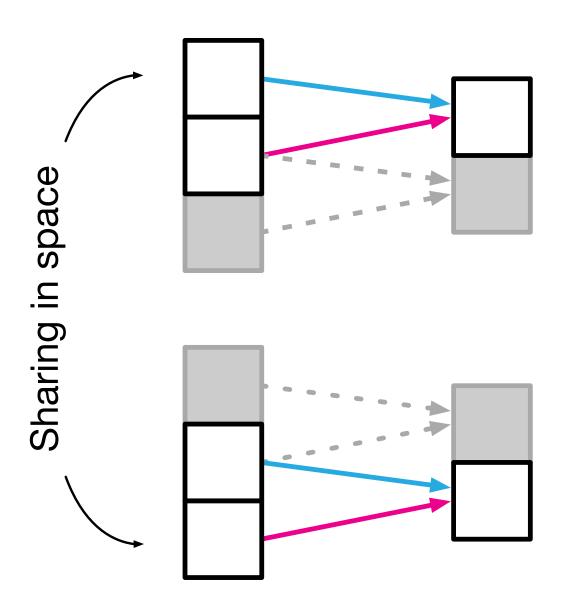


Fully-Connected Layer

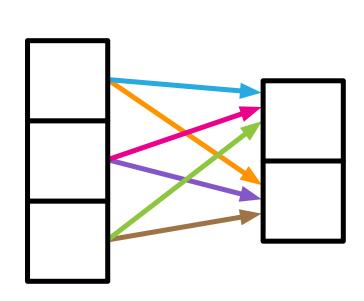
Unstructured computation
Unstructured representation



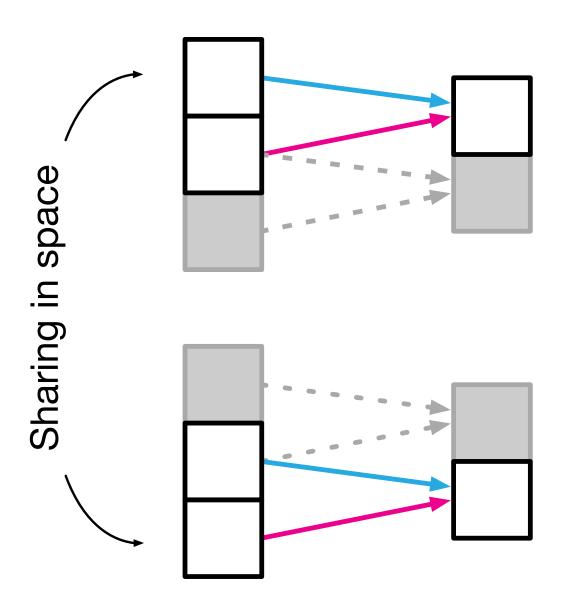
Fully-Connected Layer
Unstructured computation
Unstructured representation



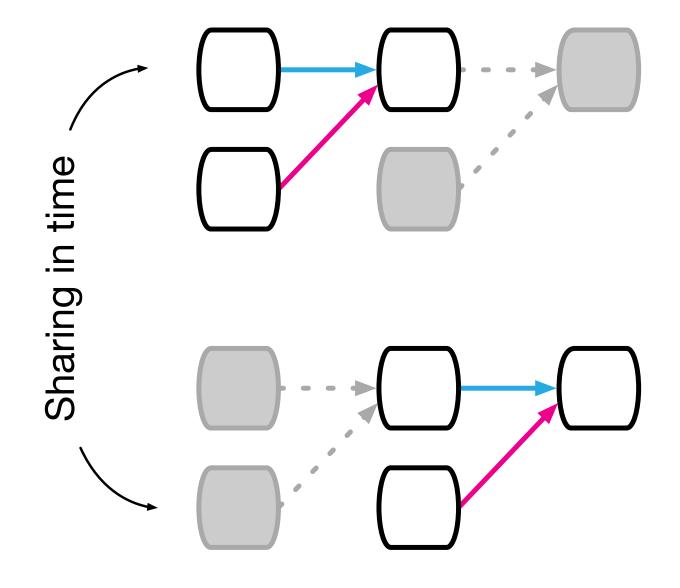
Convolutional Layer
Structured computation
Unstructured representation



Fully-Connected Layer
Unstructured computation
Unstructured representation



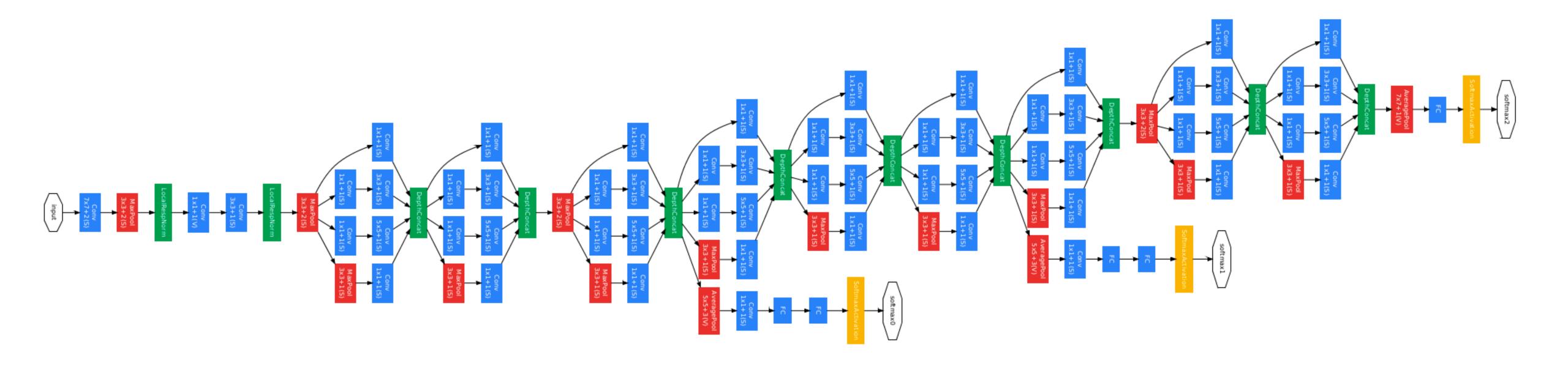
Convolutional Layer
Structured computation
Unstructured representation



Recurrent Layer

Structured computation

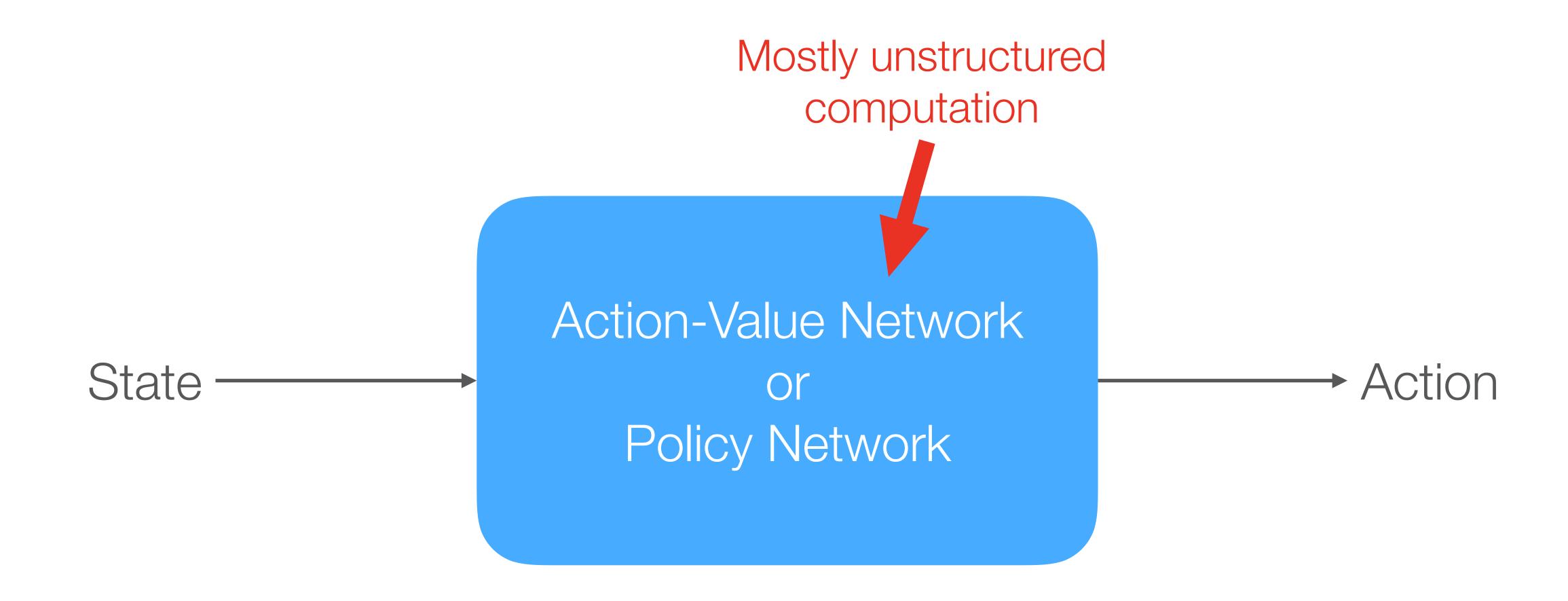
Unstructured representation



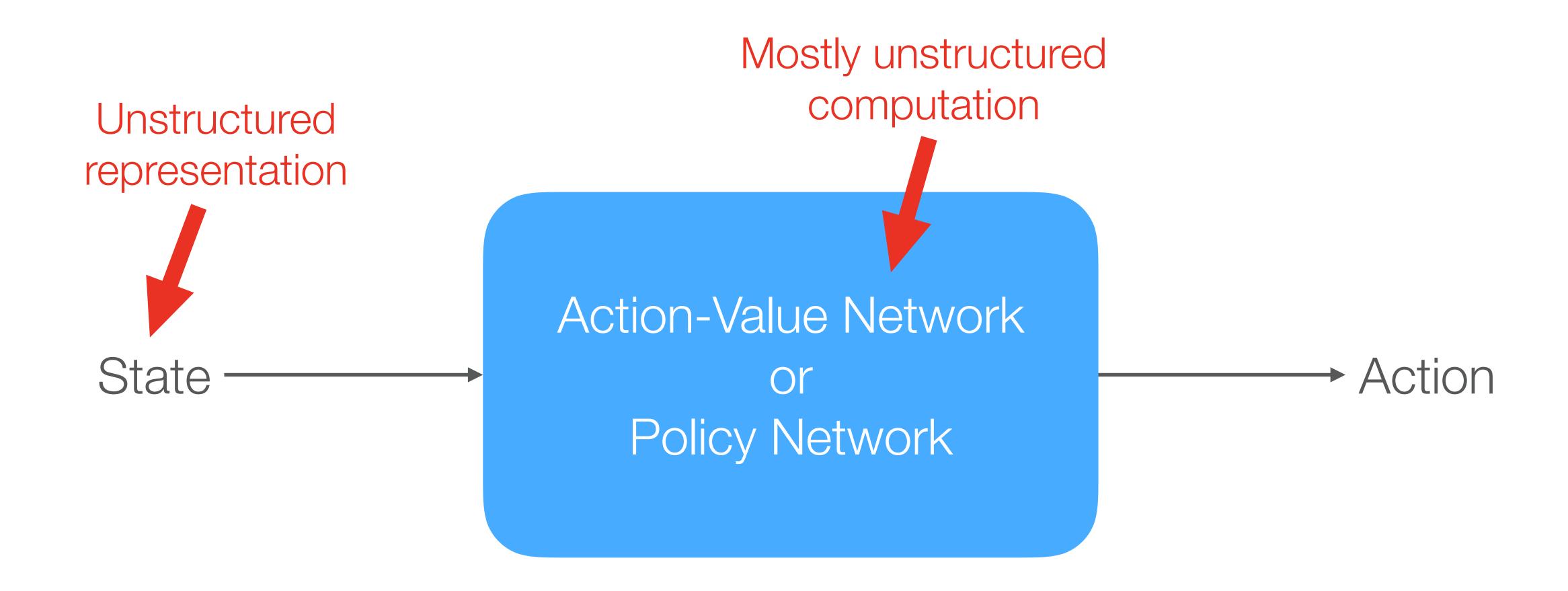
GoogLeNet with Inception modules, Szegedy et al. (2015)

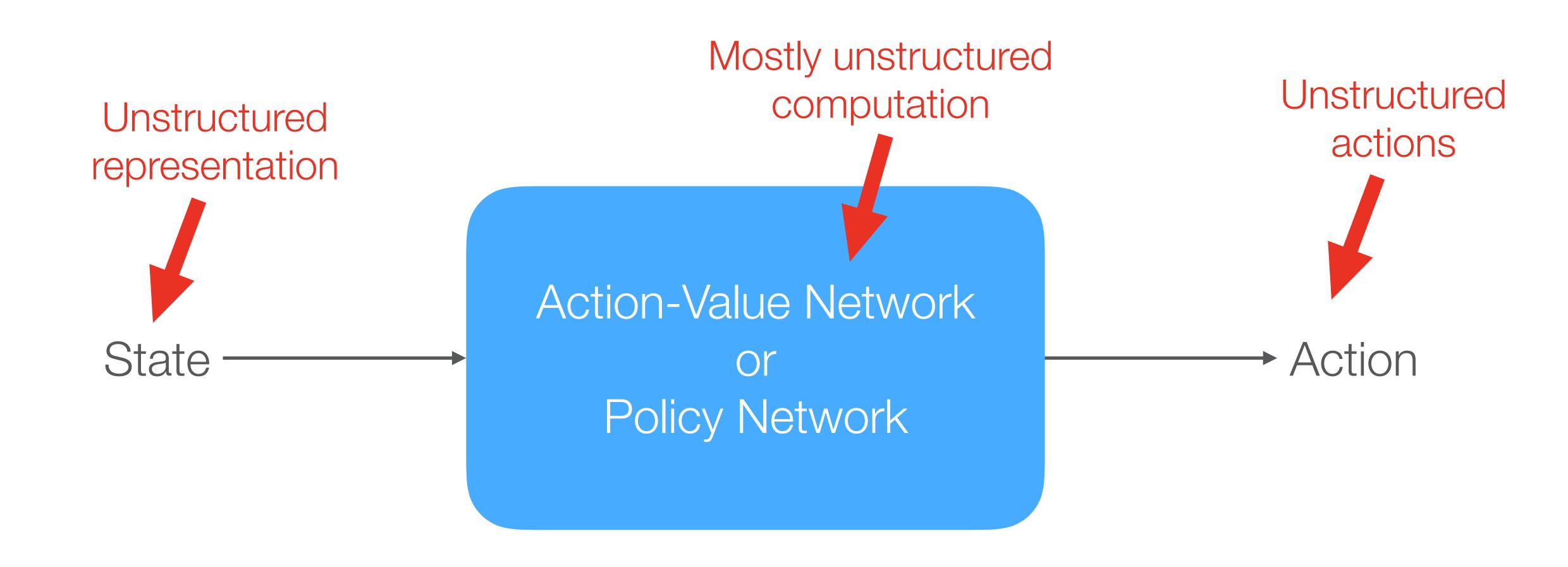




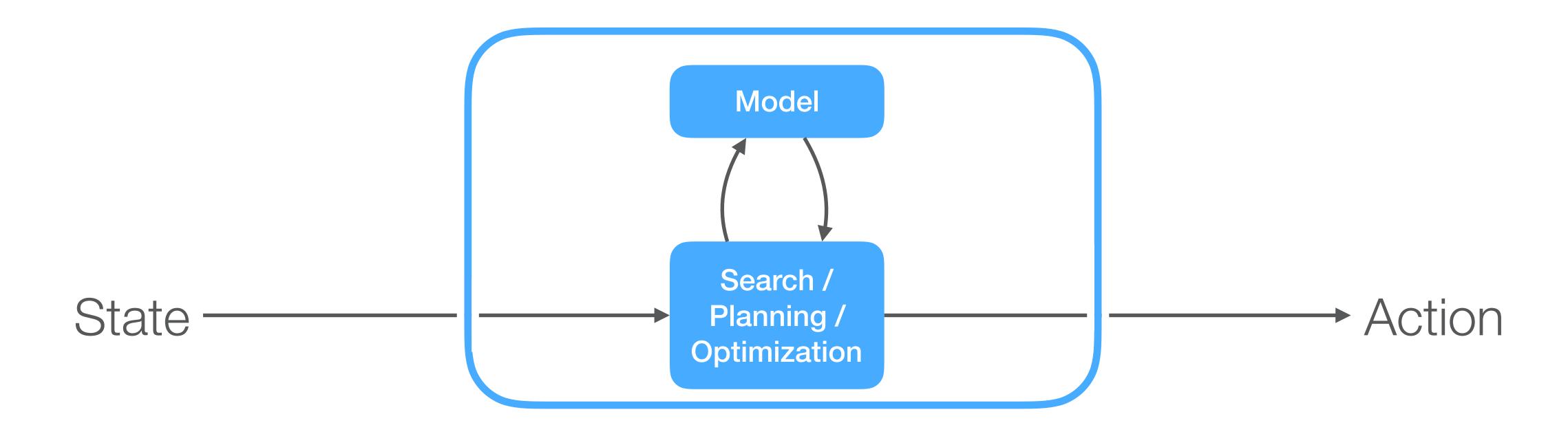






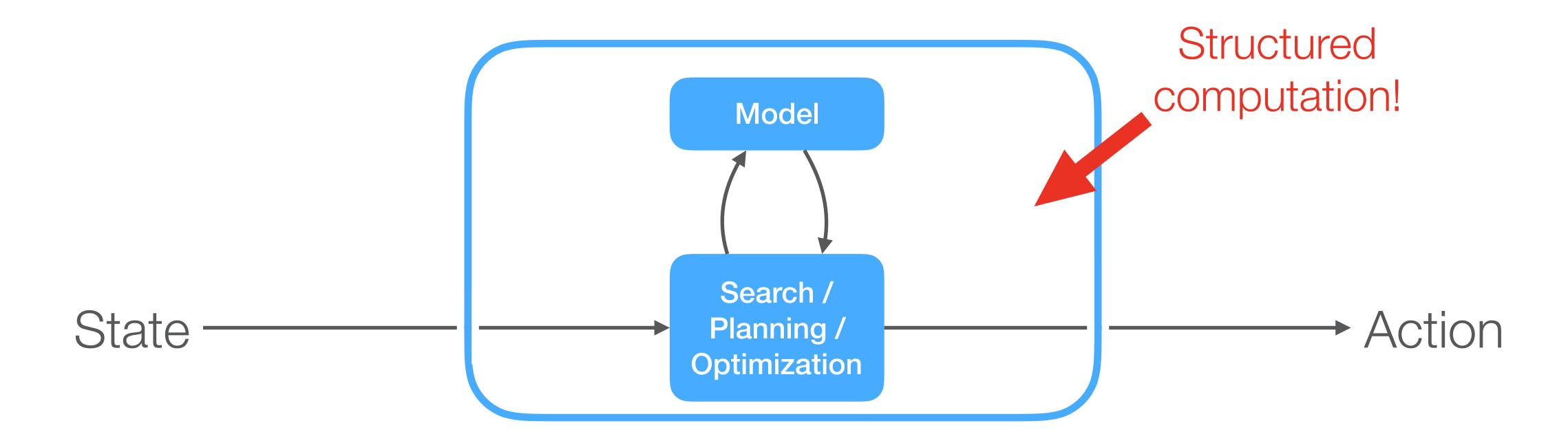


## Structure in Model-Based Deep RL





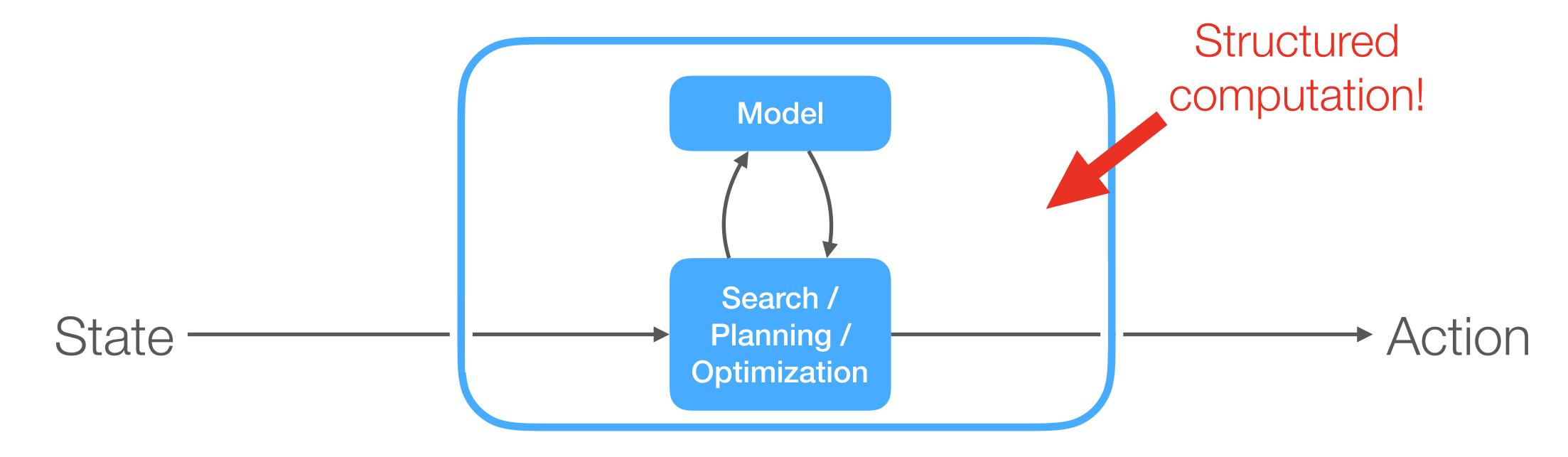
## Structure in Model-Based Deep RL





## Structure in Model-Based Deep RL

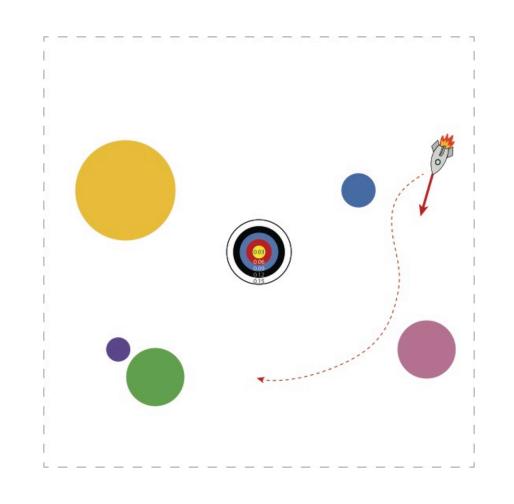
- AlphaGo (Silver et al., 2016)
- Imagination-Augmented Agents (Weber et al., 2017)
- Gradient Based Planning (Henaff et al., 2017)
- Value Prediction Networks (Oh et al., 2017)
- Universal Planning Networks (Srinivas et al., 2018)
- ... and more! See Hamrick (2019, Current Opinion in Behavioral Sciences) for a review.



#### Outline

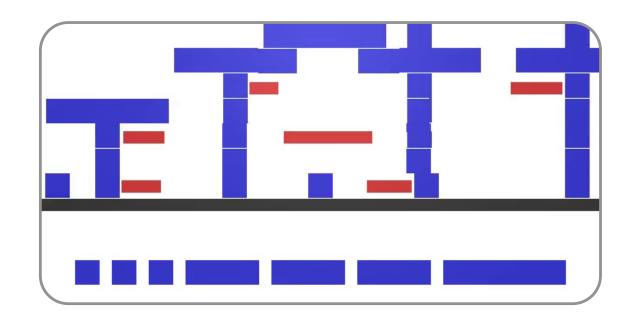
1. **Structured Computation**: the way that individual computations or functions are composed into more complex structures.

Hamrick, Ballard, Pascanu, Vinyals, Heess, & Battaglia (2017, ICLR)



2. Structured Representation: the format of the data that computations are performed over.

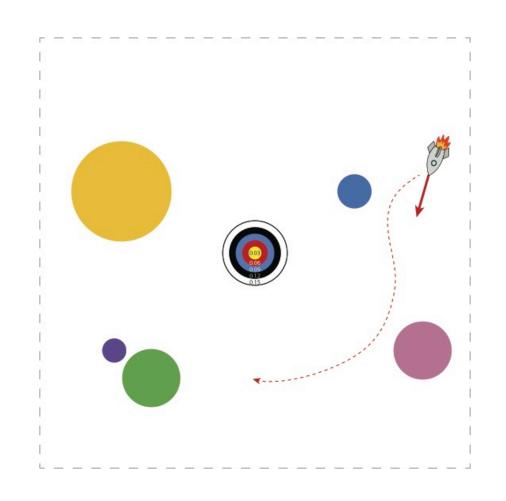
Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018, arXiv) Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia, & Hamrick (2019, ICML)



#### Outline

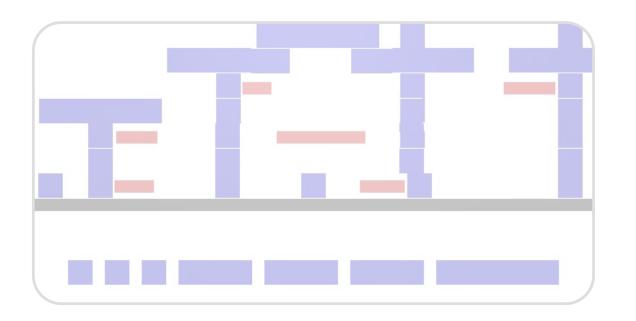
1. **Structured Computation**: the way that individual computations or functions are composed into more complex structures.

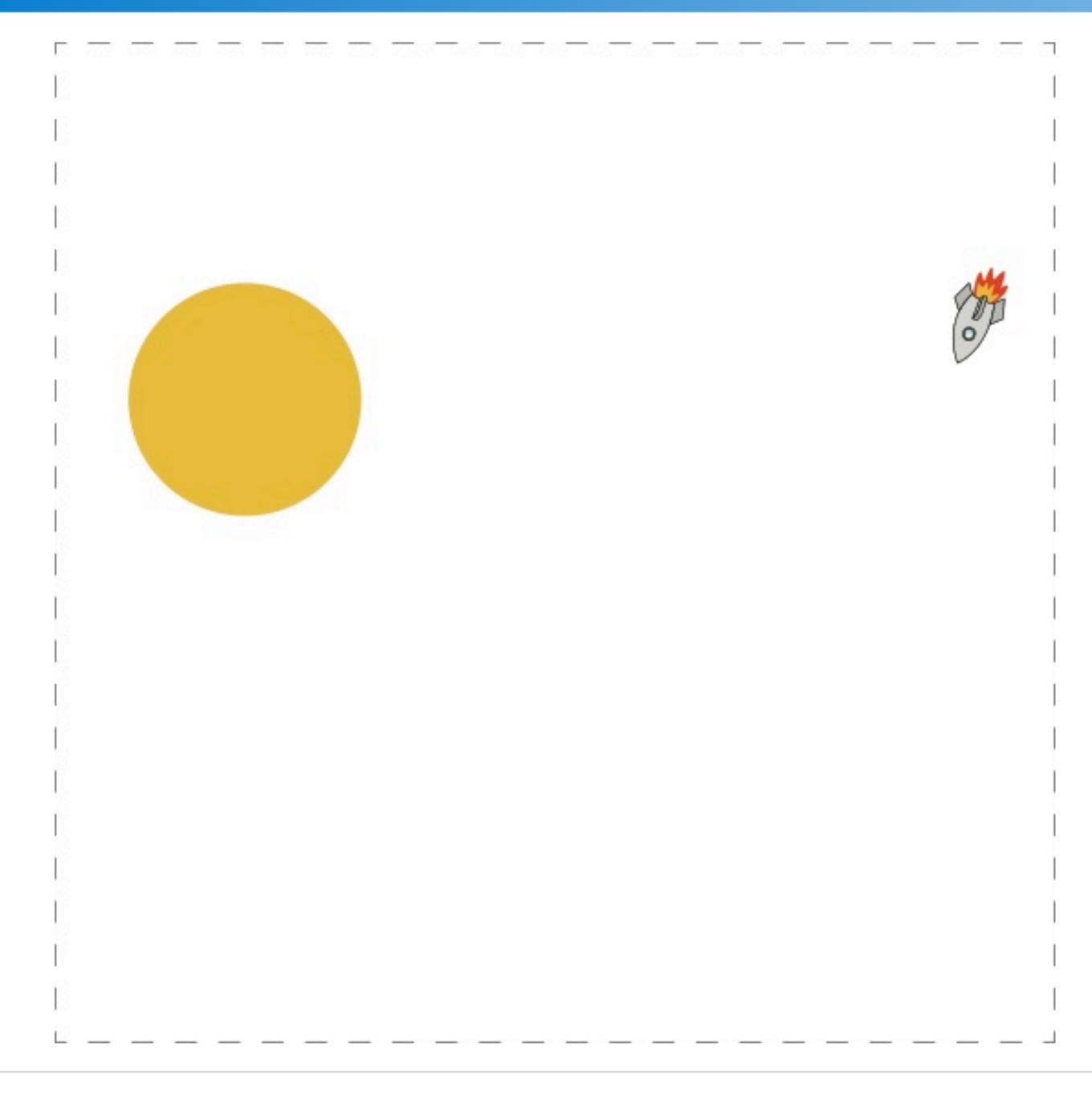
Hamrick, Ballard, Pascanu, Vinyals, Heess, & Battaglia (2017, ICLR)

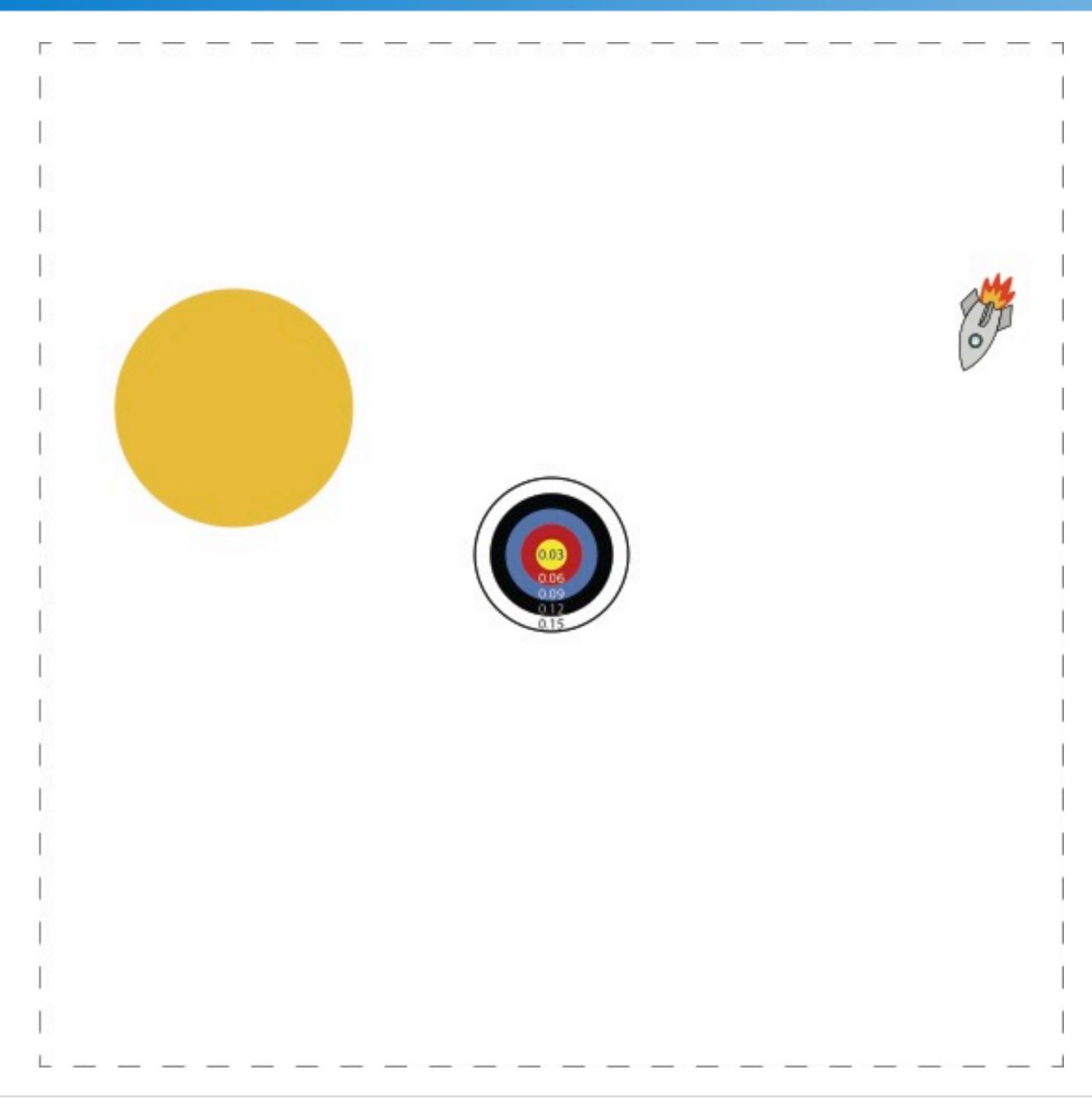


2. Structured Representation: the format of the data that computations are performed over.

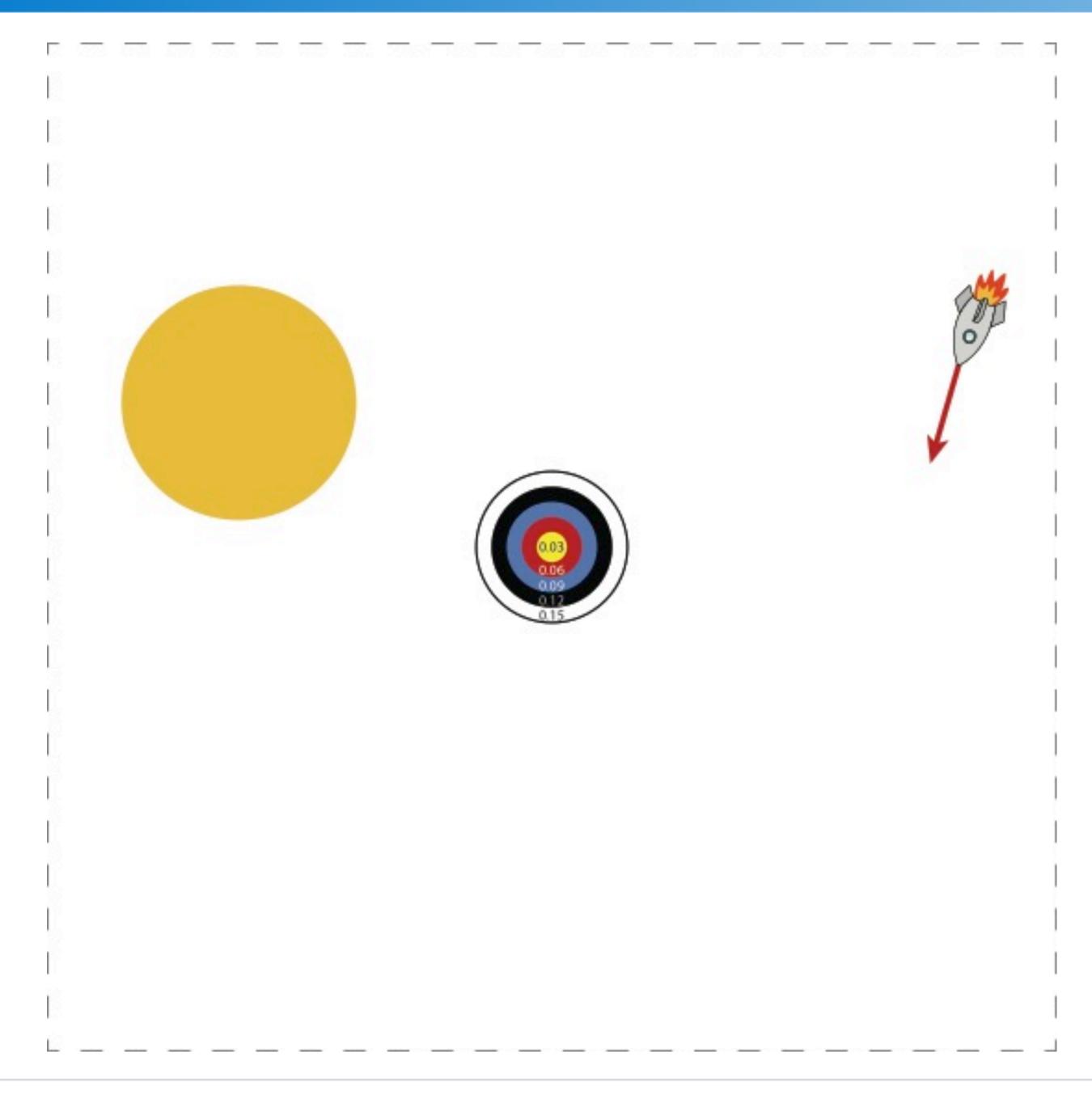
Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018, arXiv) Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia, & Hamrick (2019, ICML)



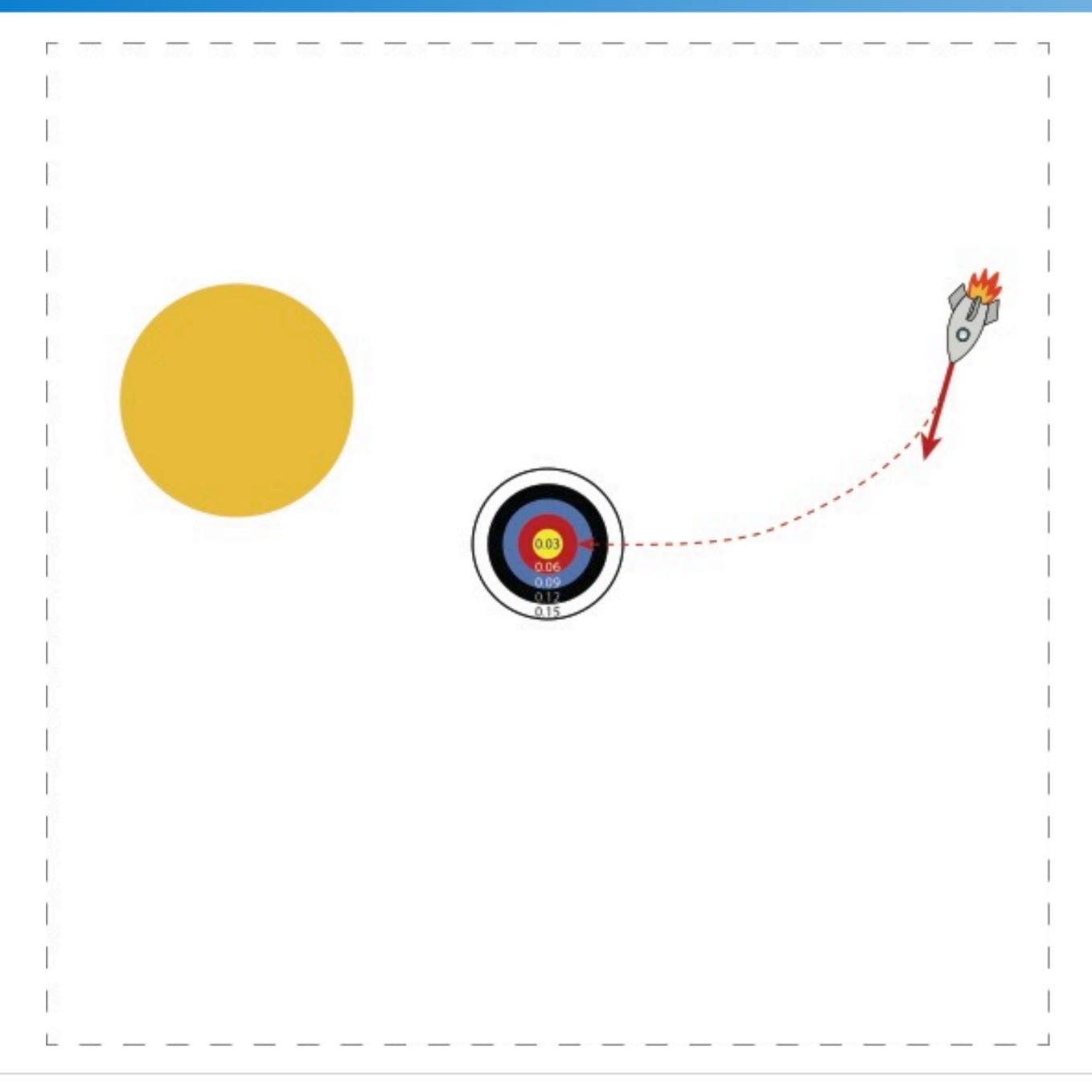


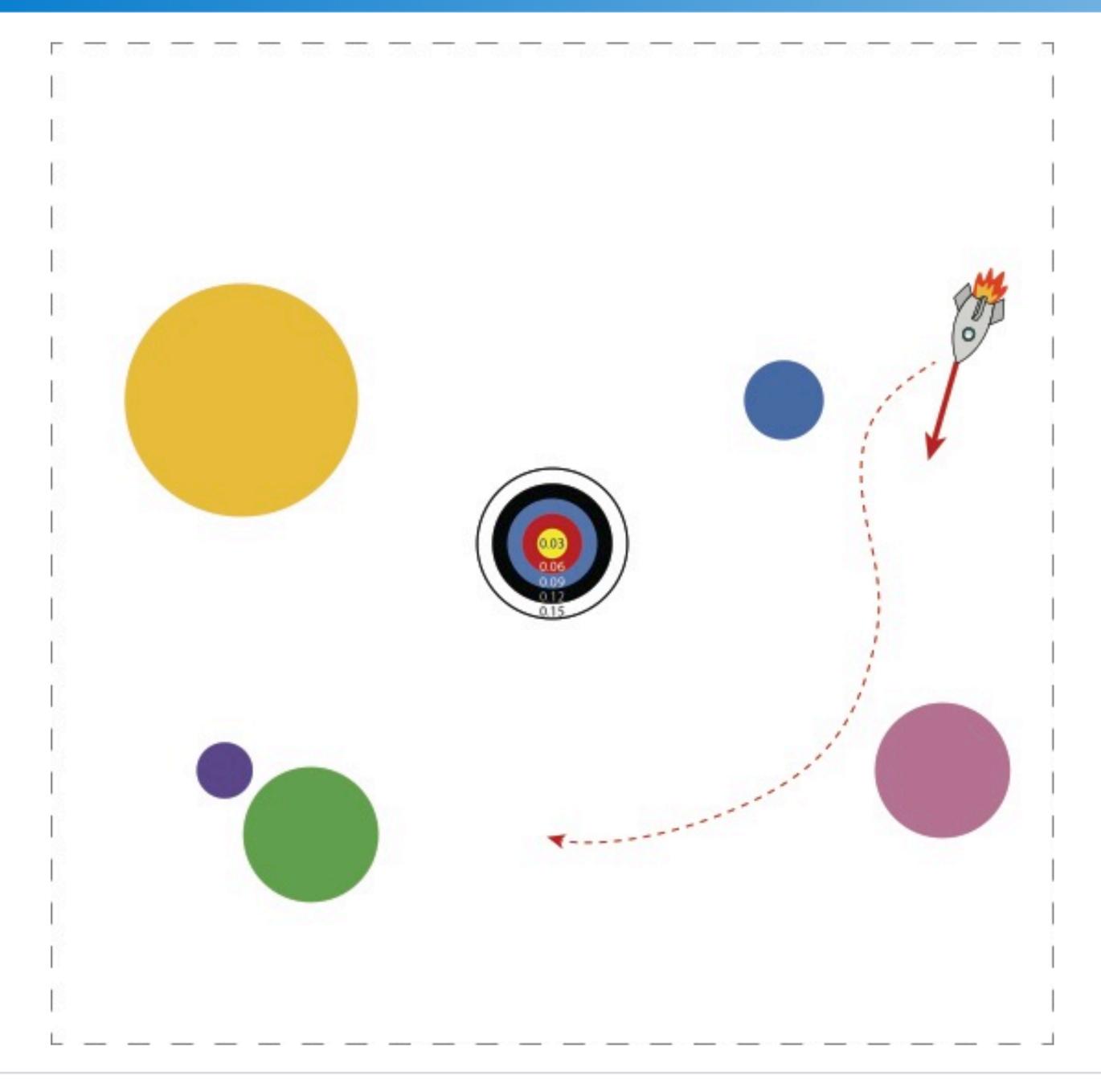




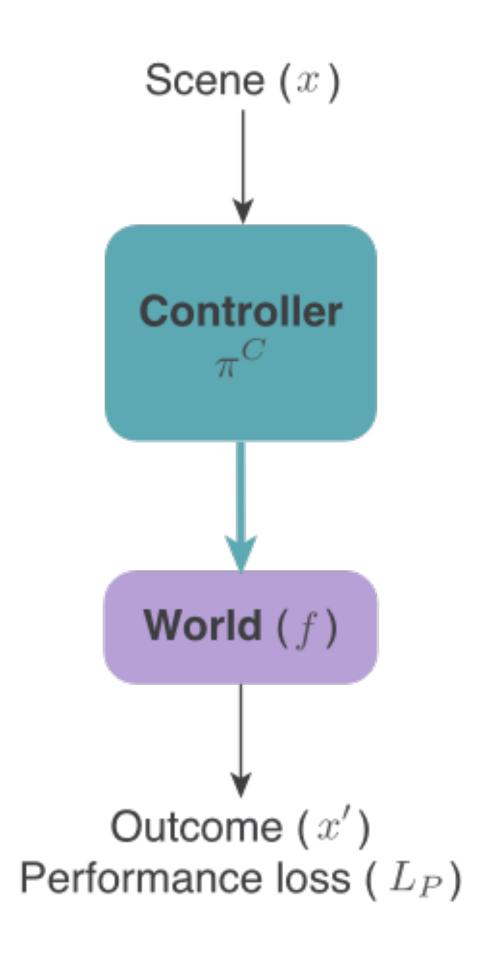








## Reactive Controller

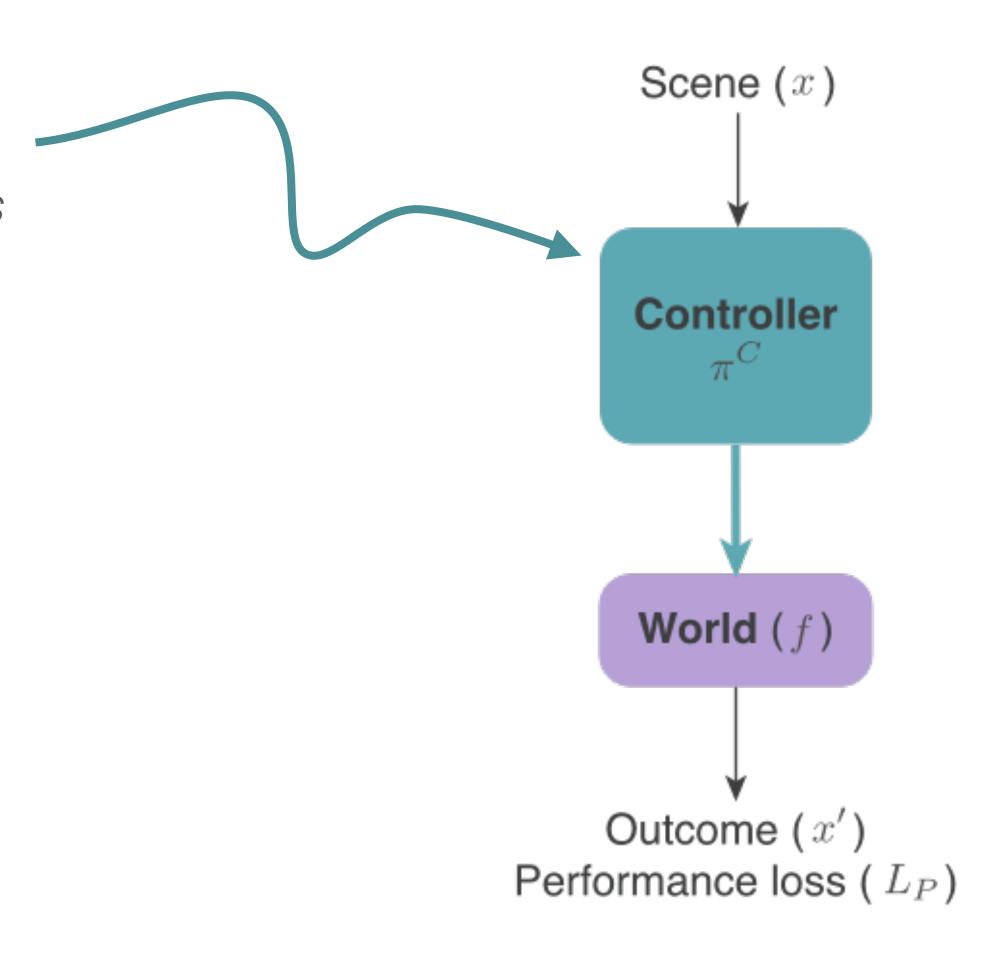




## Reactive Controller

#### Controller (MLP):

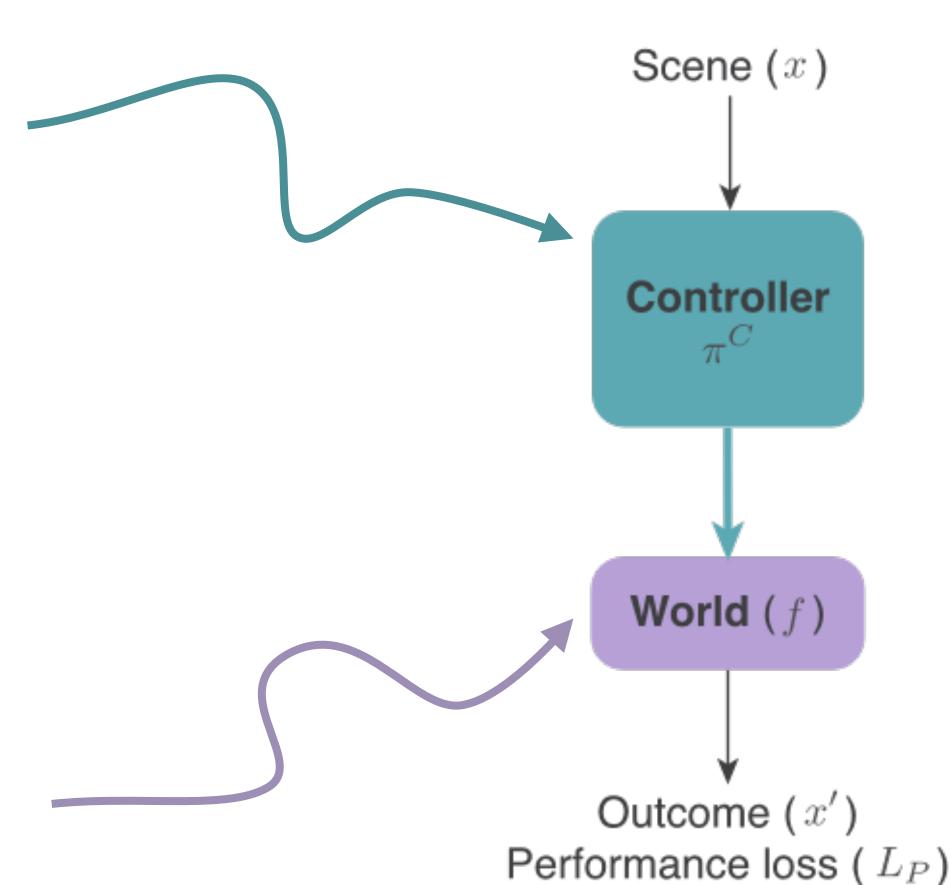
Proposes a control that is sent to the world to minimize performance loss



### Reactive Controller

#### Controller (MLP):

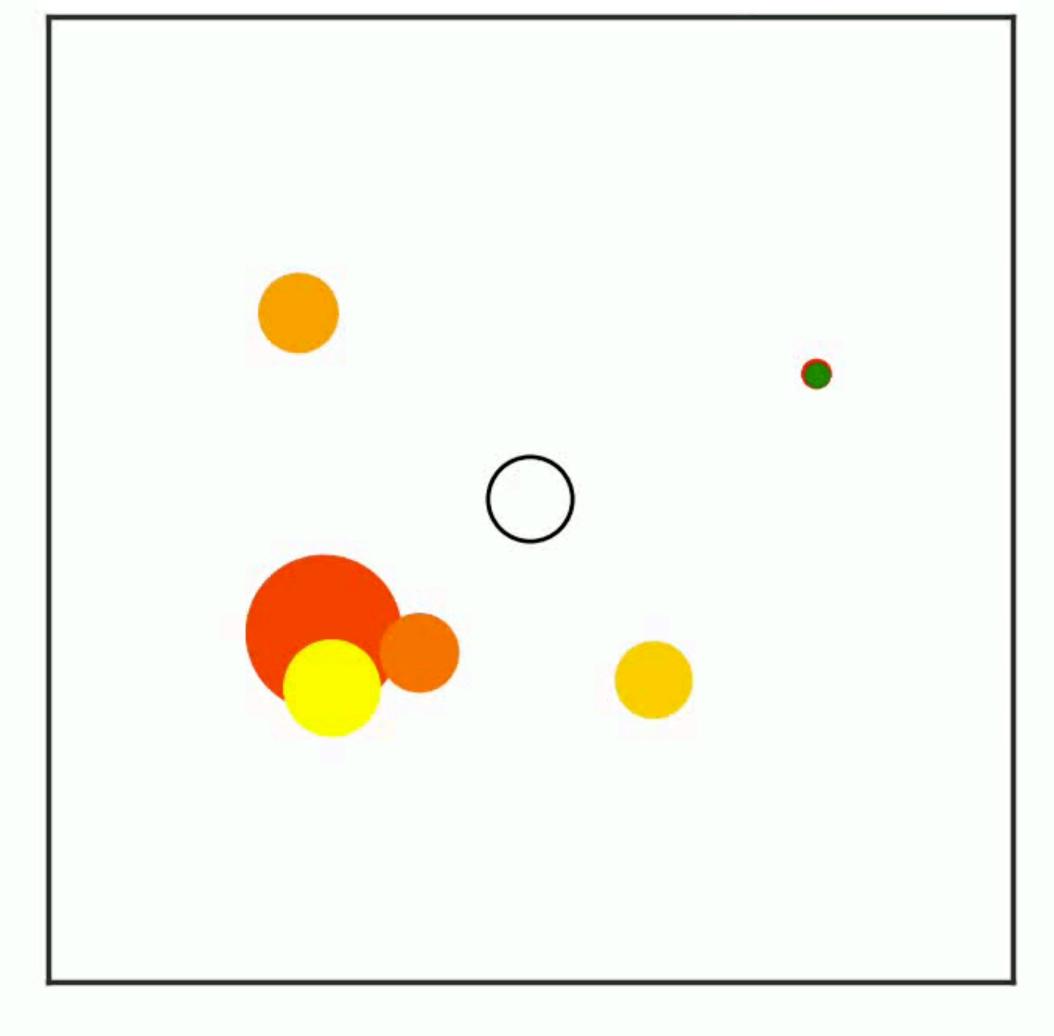
Proposes a control that is sent to the world to minimize performance loss



World: The true environment that the agent is acting in

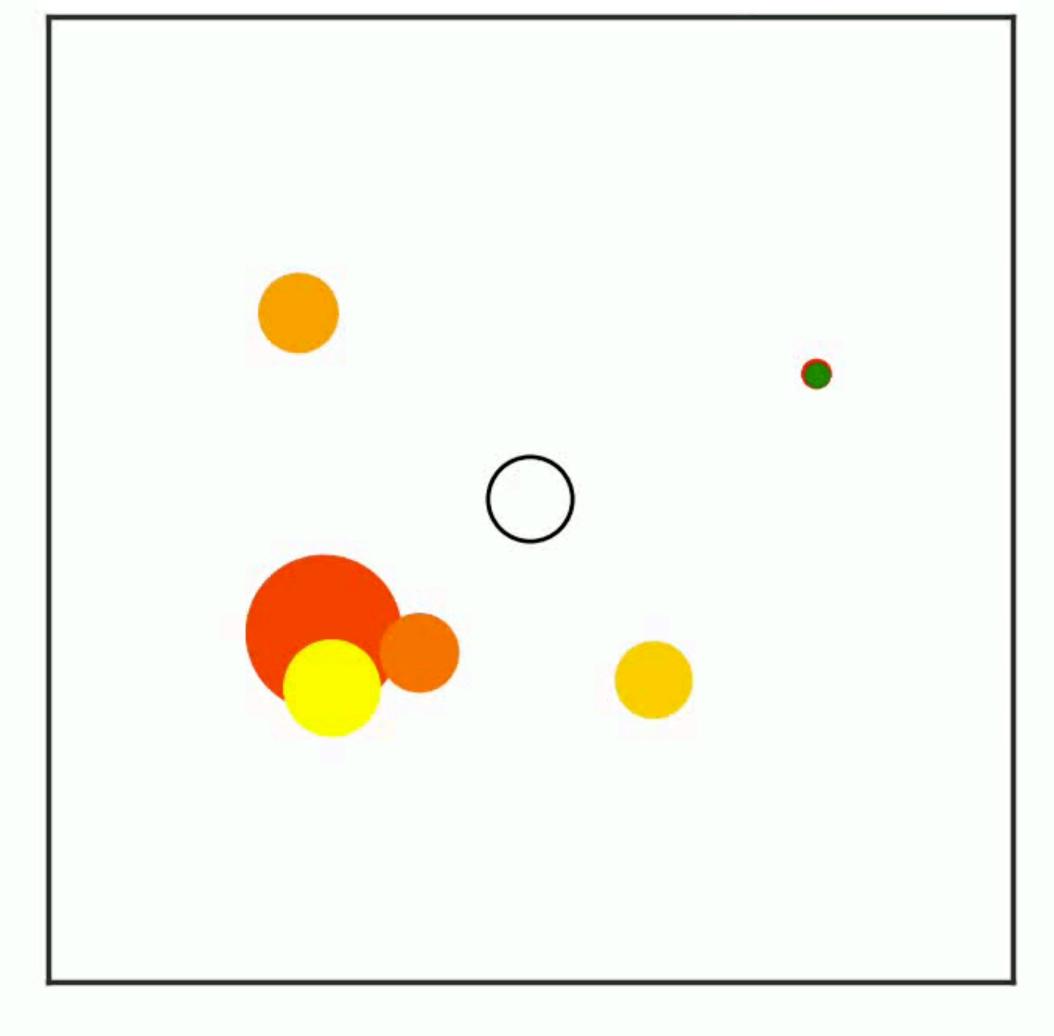


Reactive controller: simple task



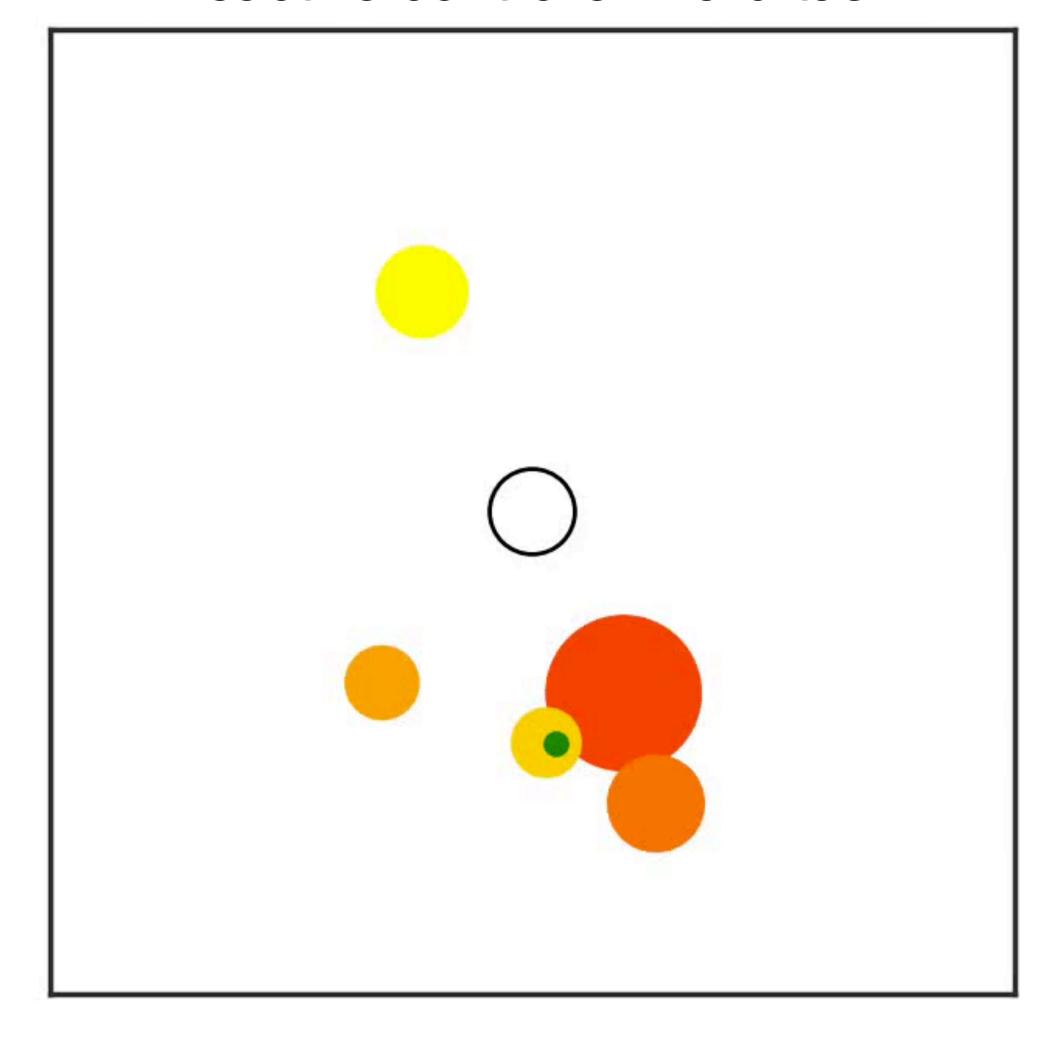


Reactive controller: simple task

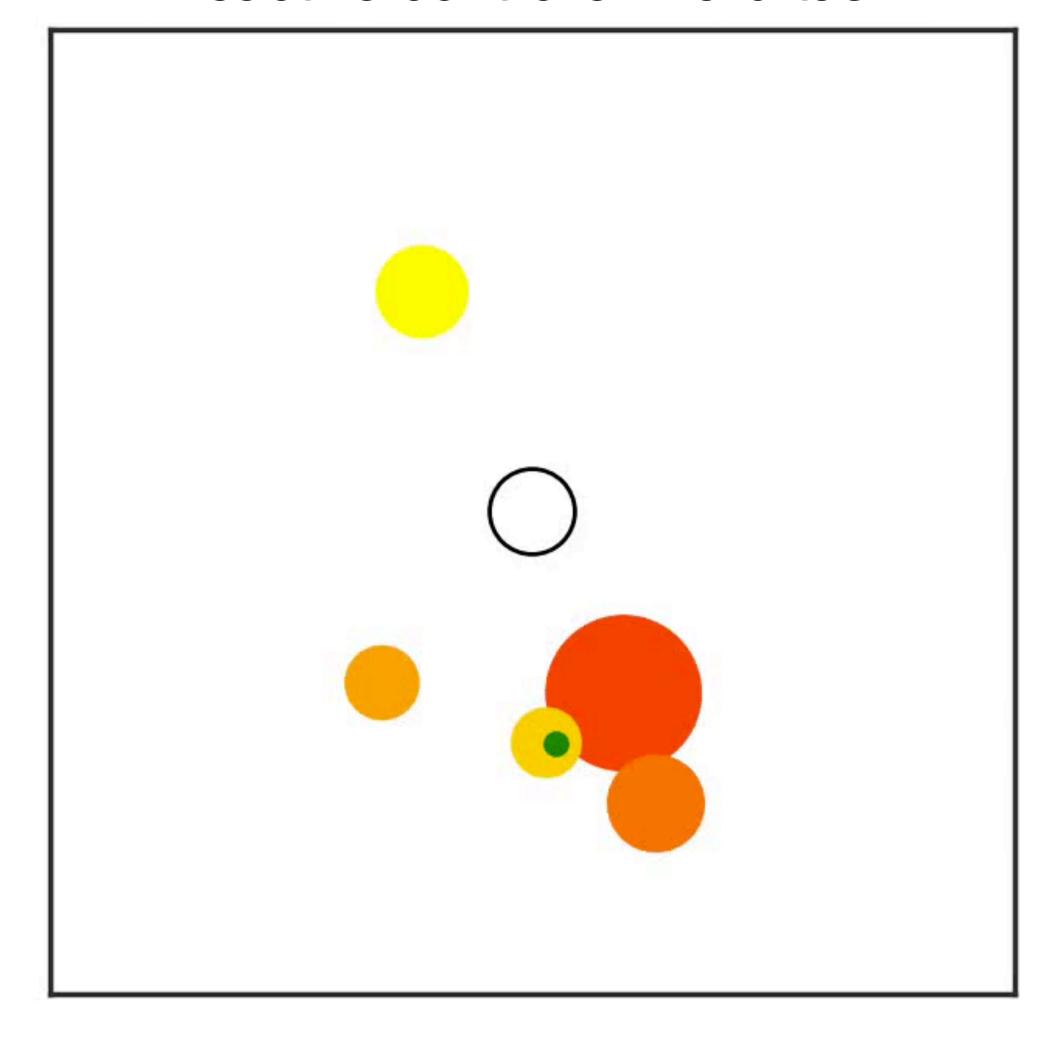


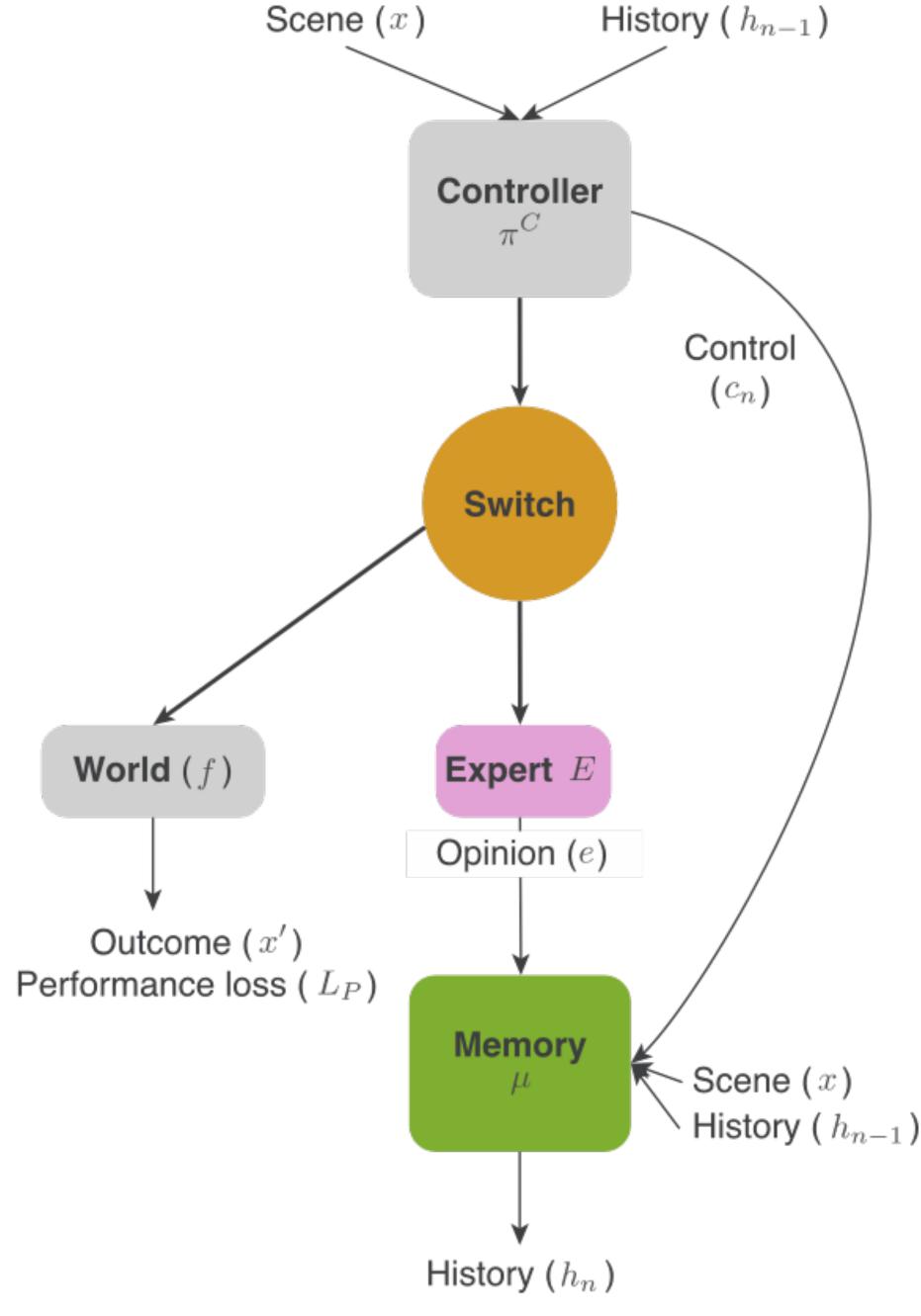


#### Reactive controller: hard task



#### Reactive controller: hard task

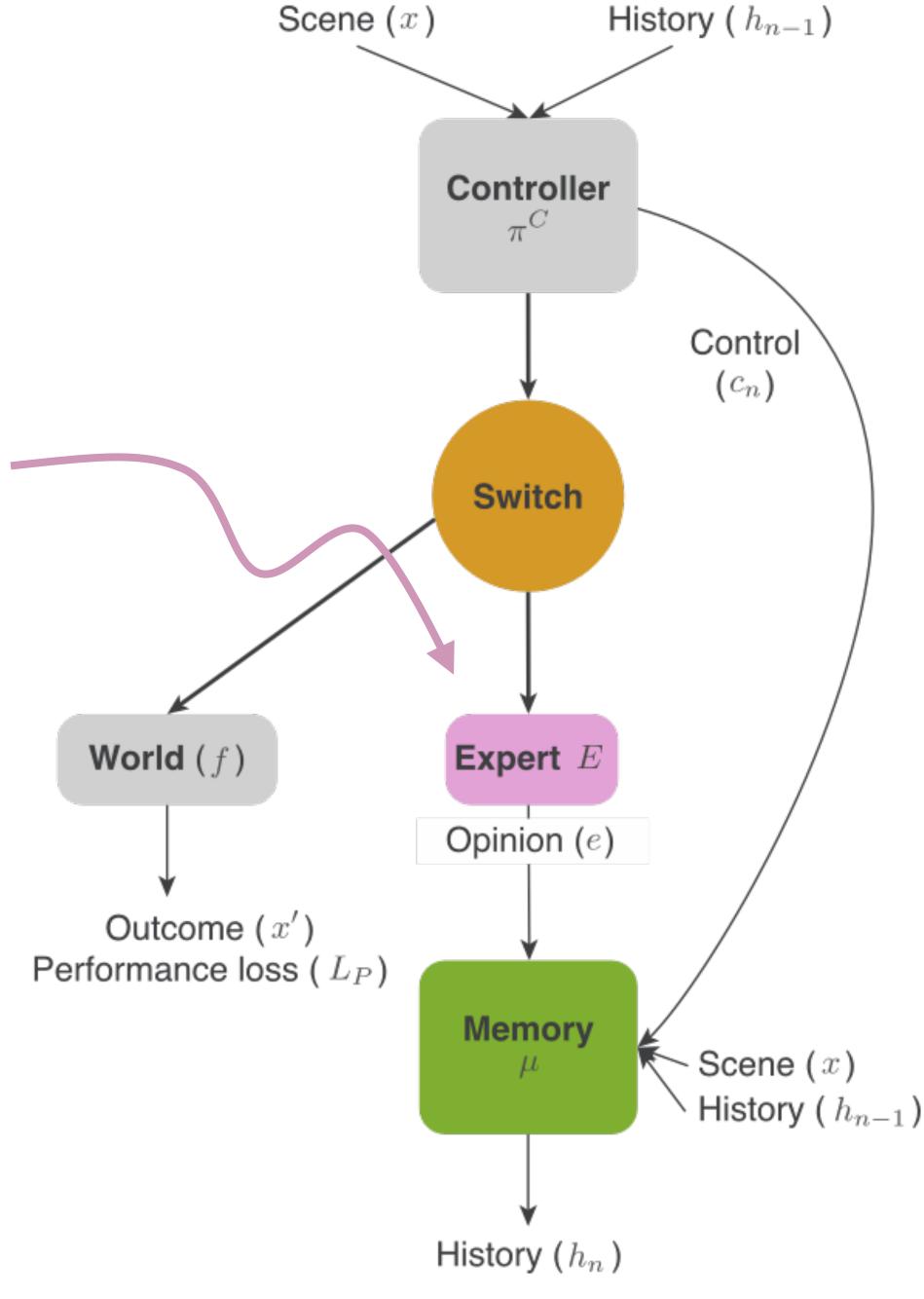






#### Expert:

Model of the world that evaluates proposed controls

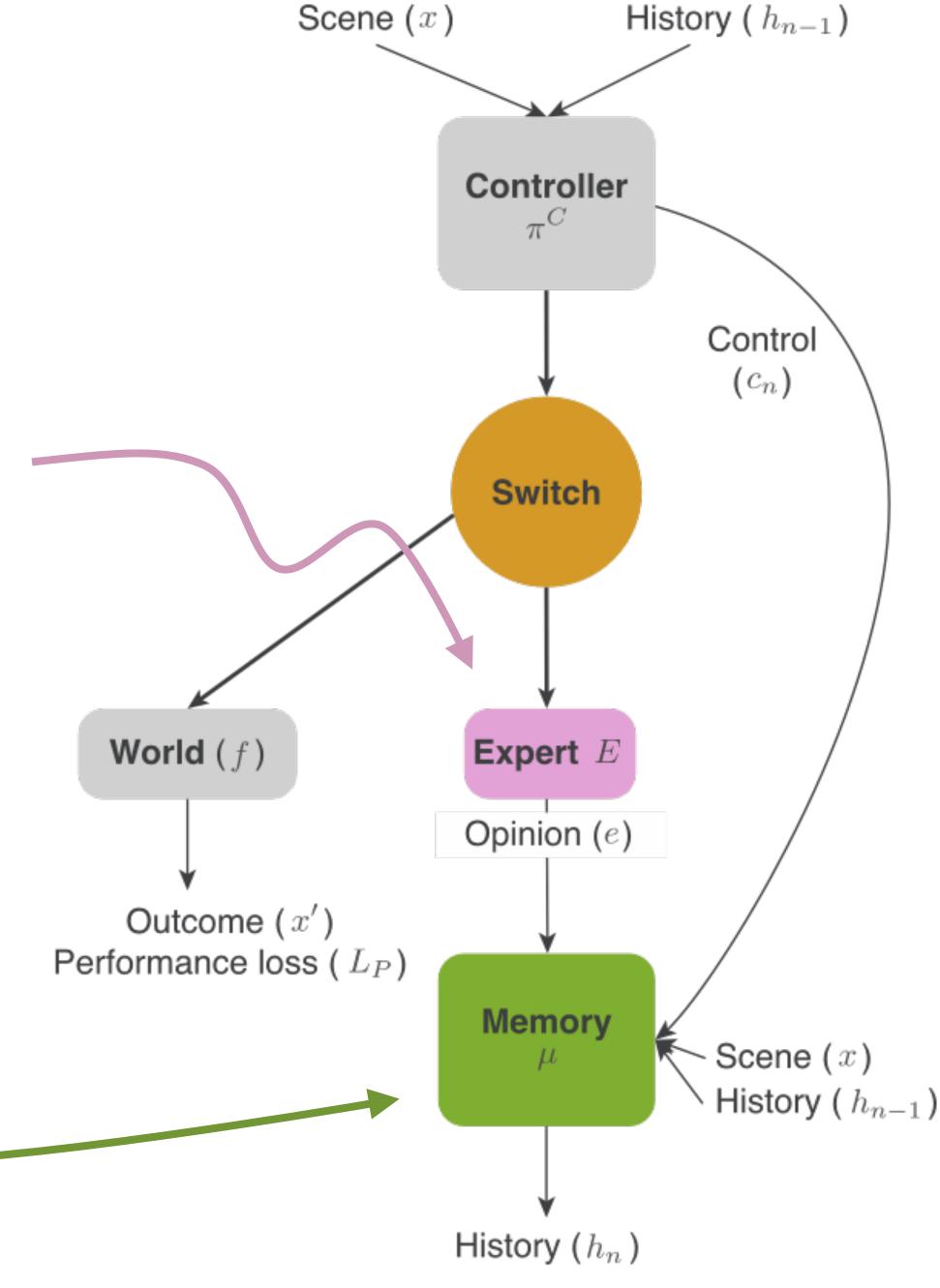




#### Expert:

Model of the world that evaluates proposed controls

#### Memory (LSTM):

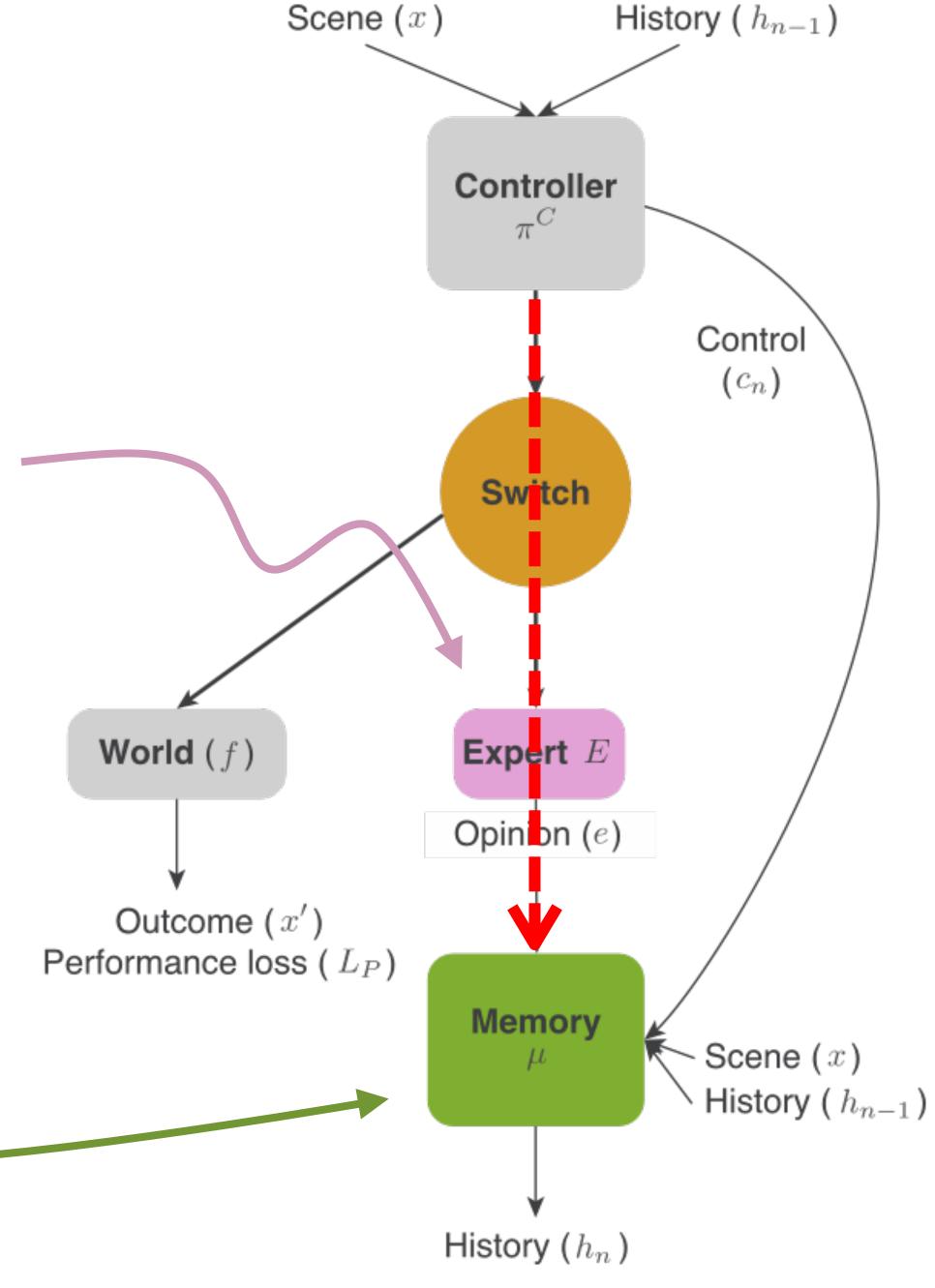




#### Expert:

Model of the world that evaluates proposed controls

#### Memory (LSTM):

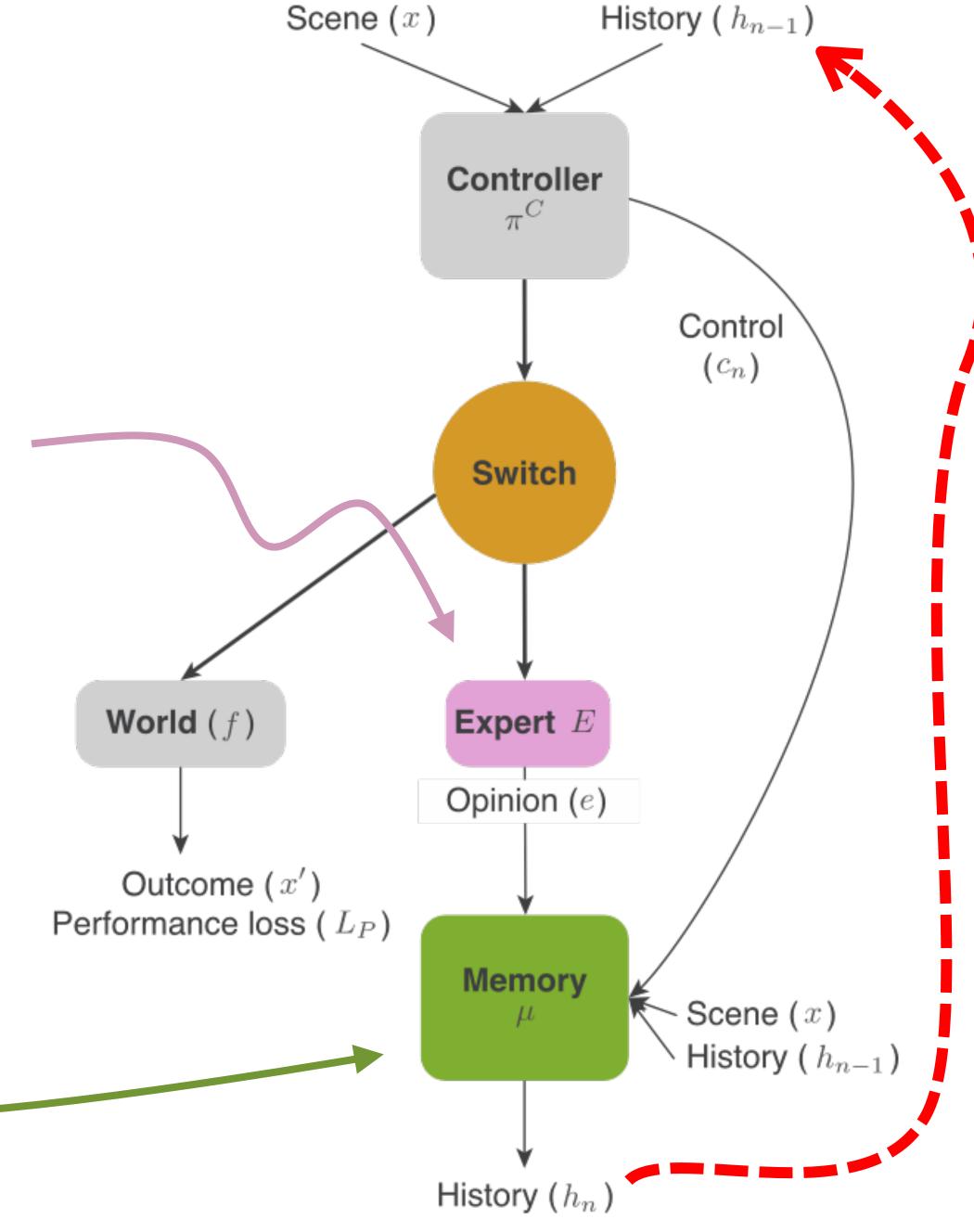




#### Expert:

Model of the world that evaluates proposed controls

#### Memory (LSTM):

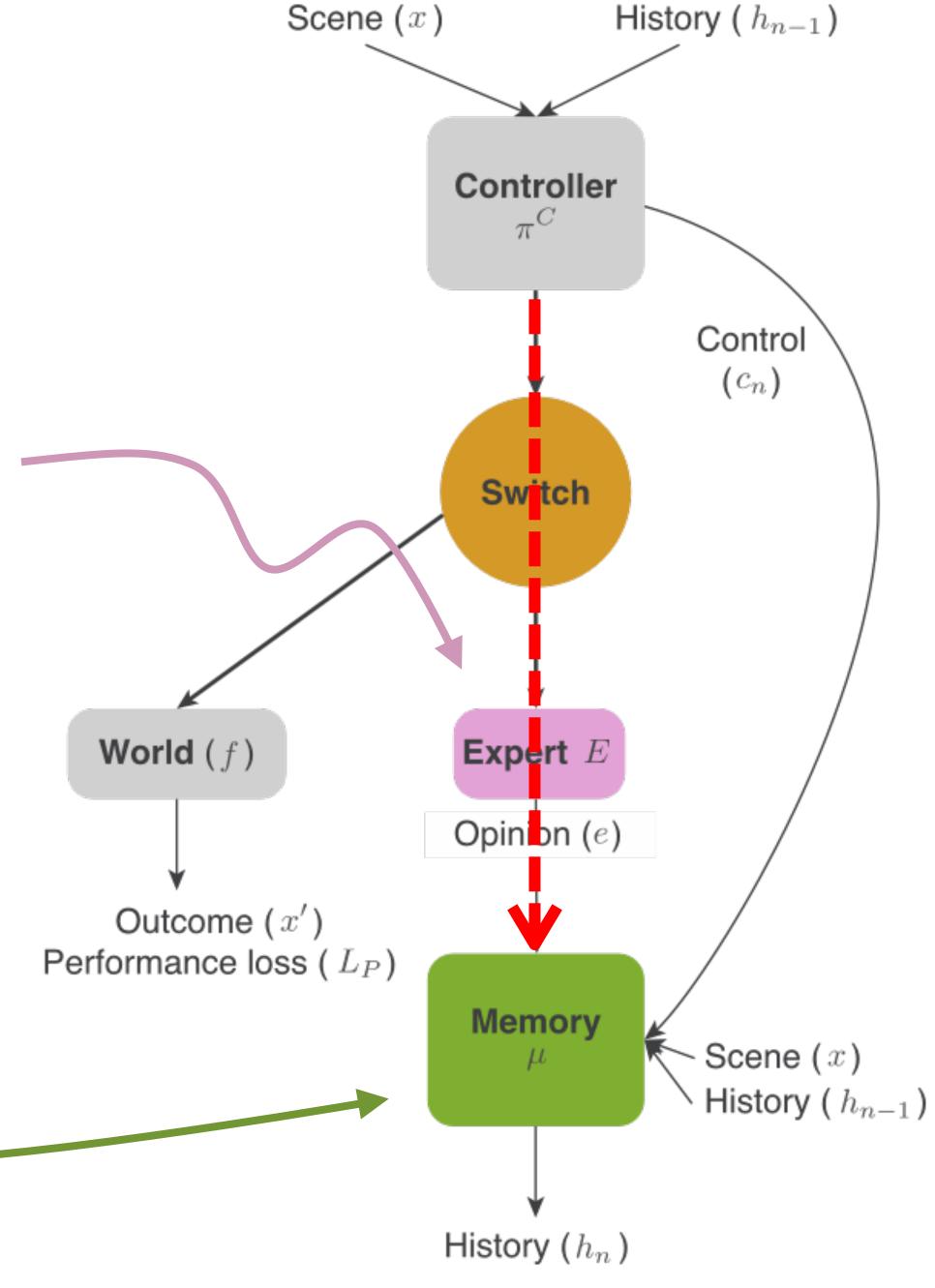




#### Expert:

Model of the world that evaluates proposed controls

#### Memory (LSTM):

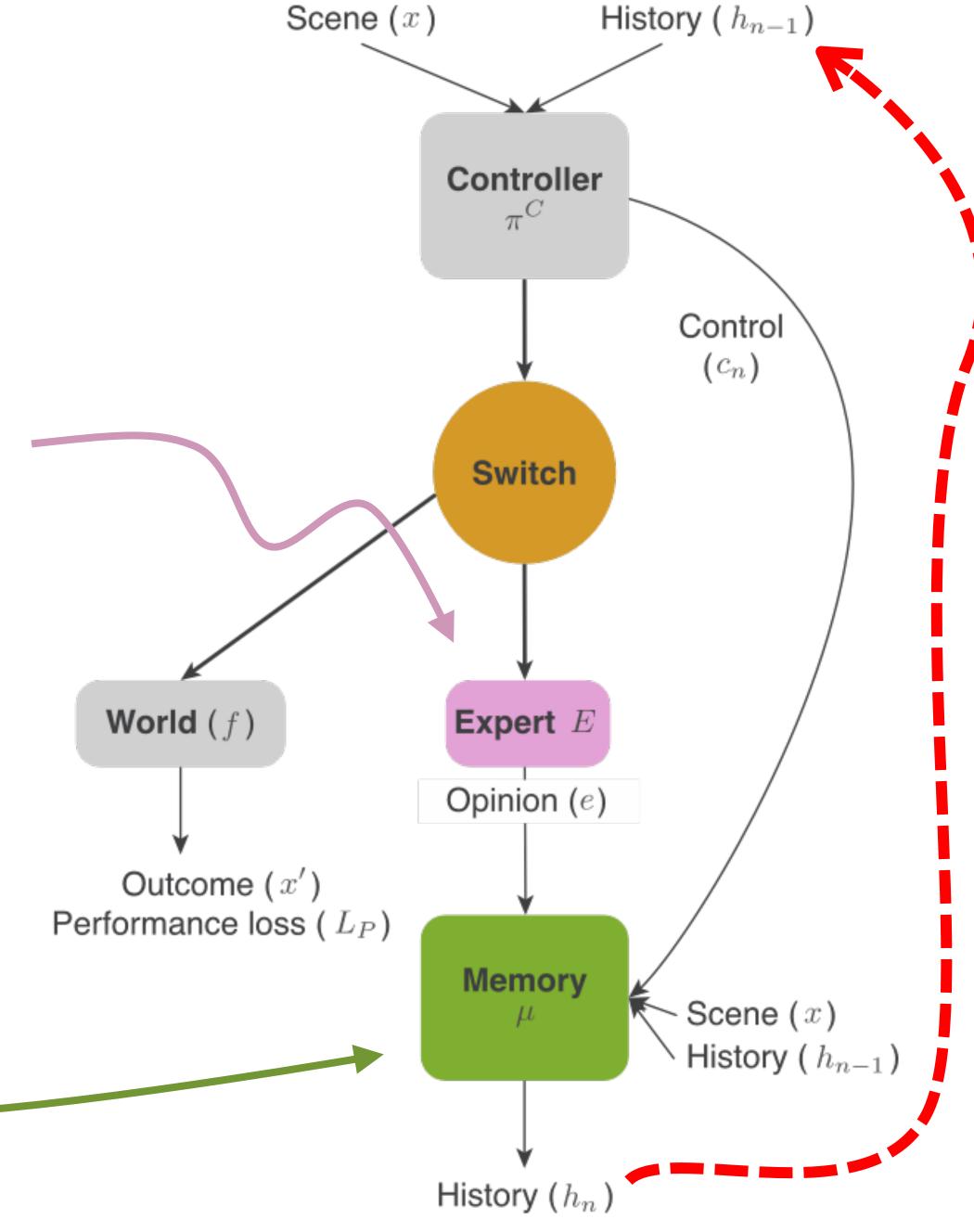




#### Expert:

Model of the world that evaluates proposed controls

#### Memory (LSTM):

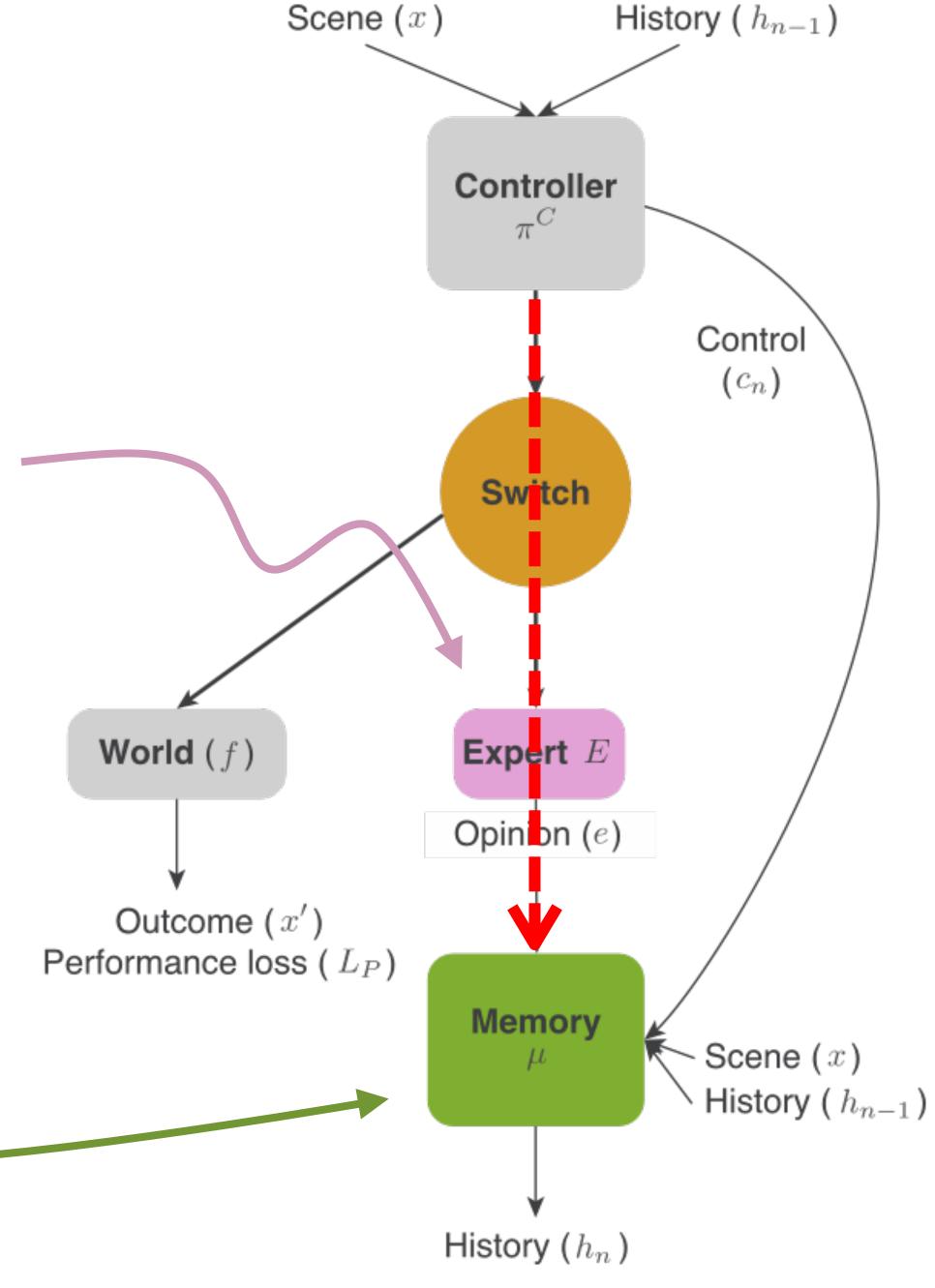




#### Expert:

Model of the world that evaluates proposed controls

#### Memory (LSTM):

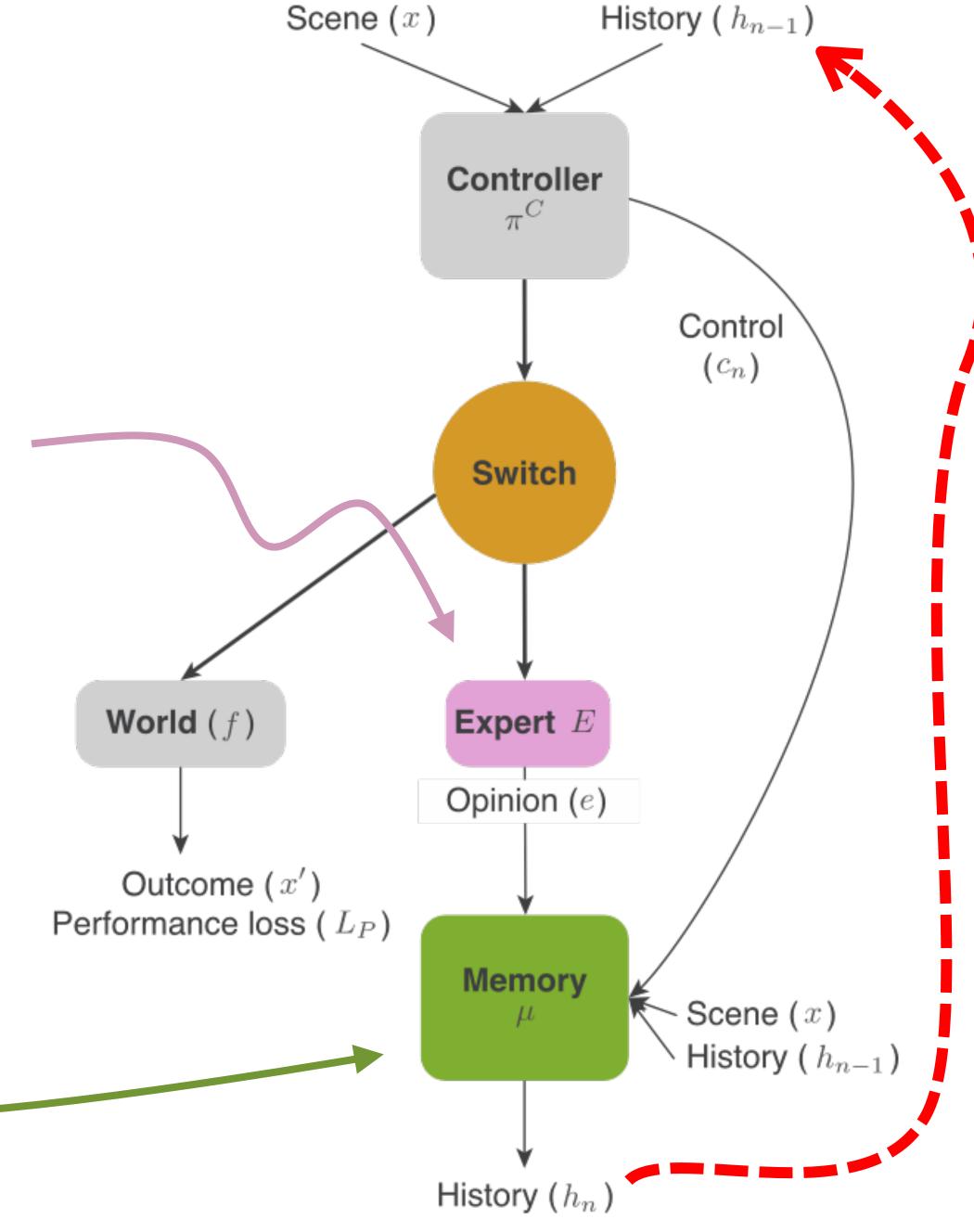




#### Expert:

Model of the world that evaluates proposed controls

#### Memory (LSTM):

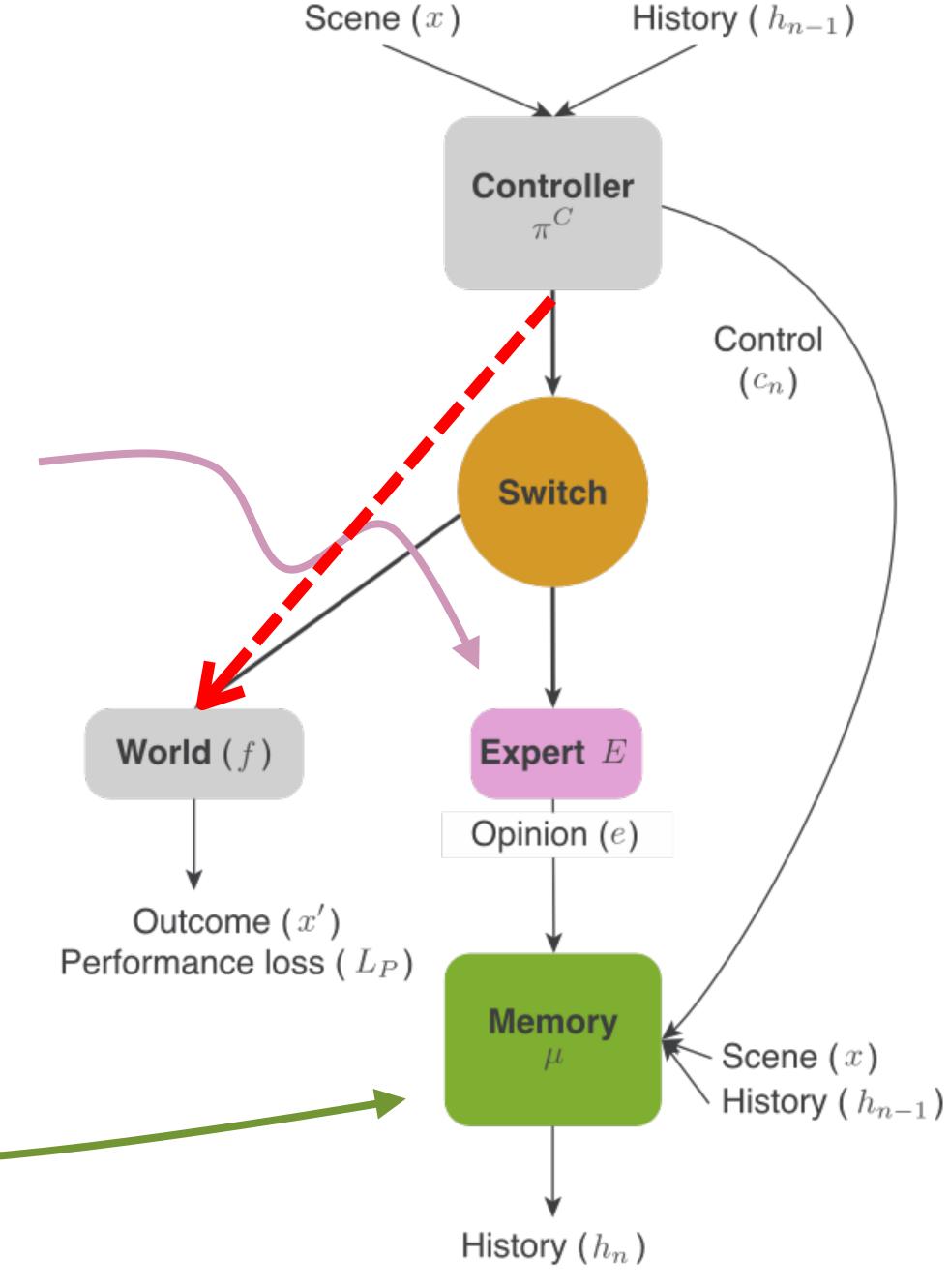




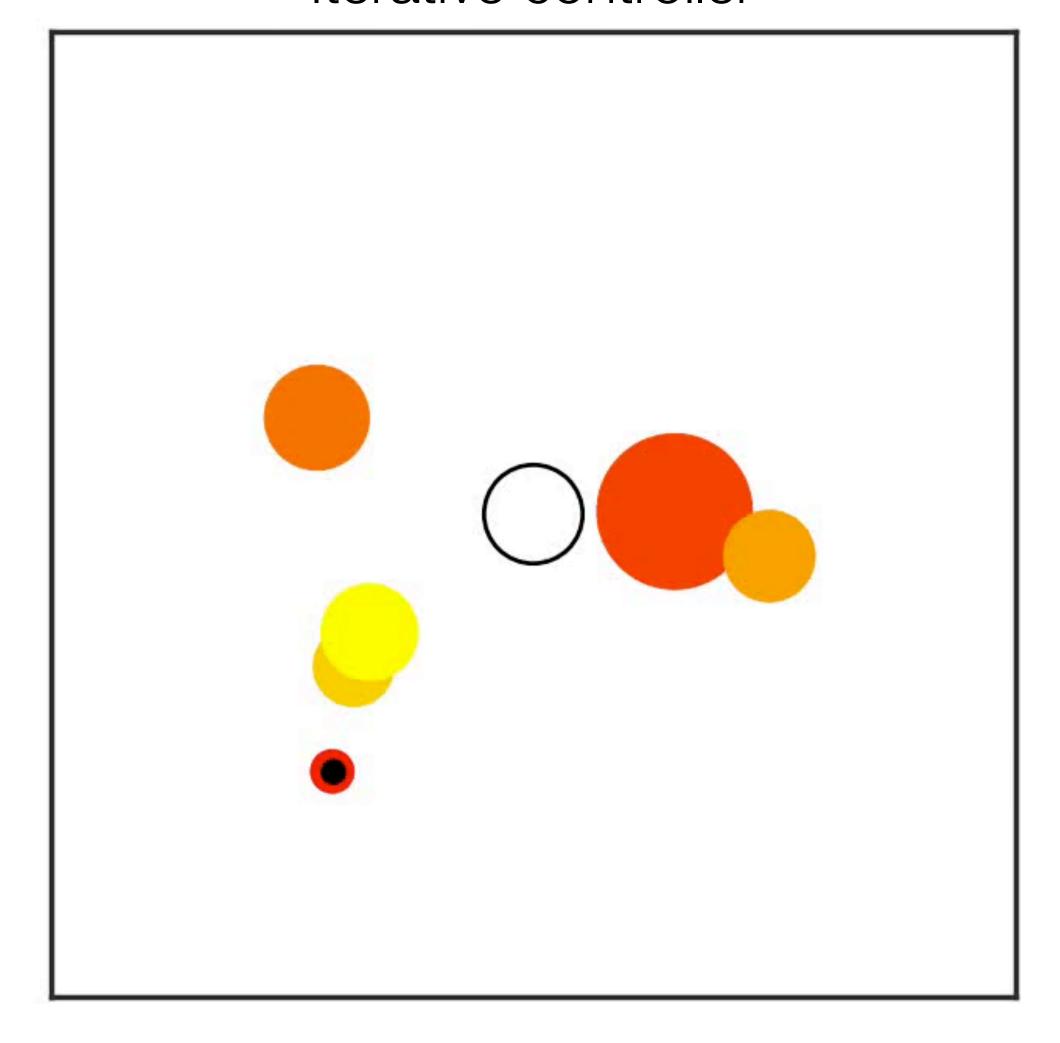
#### Expert:

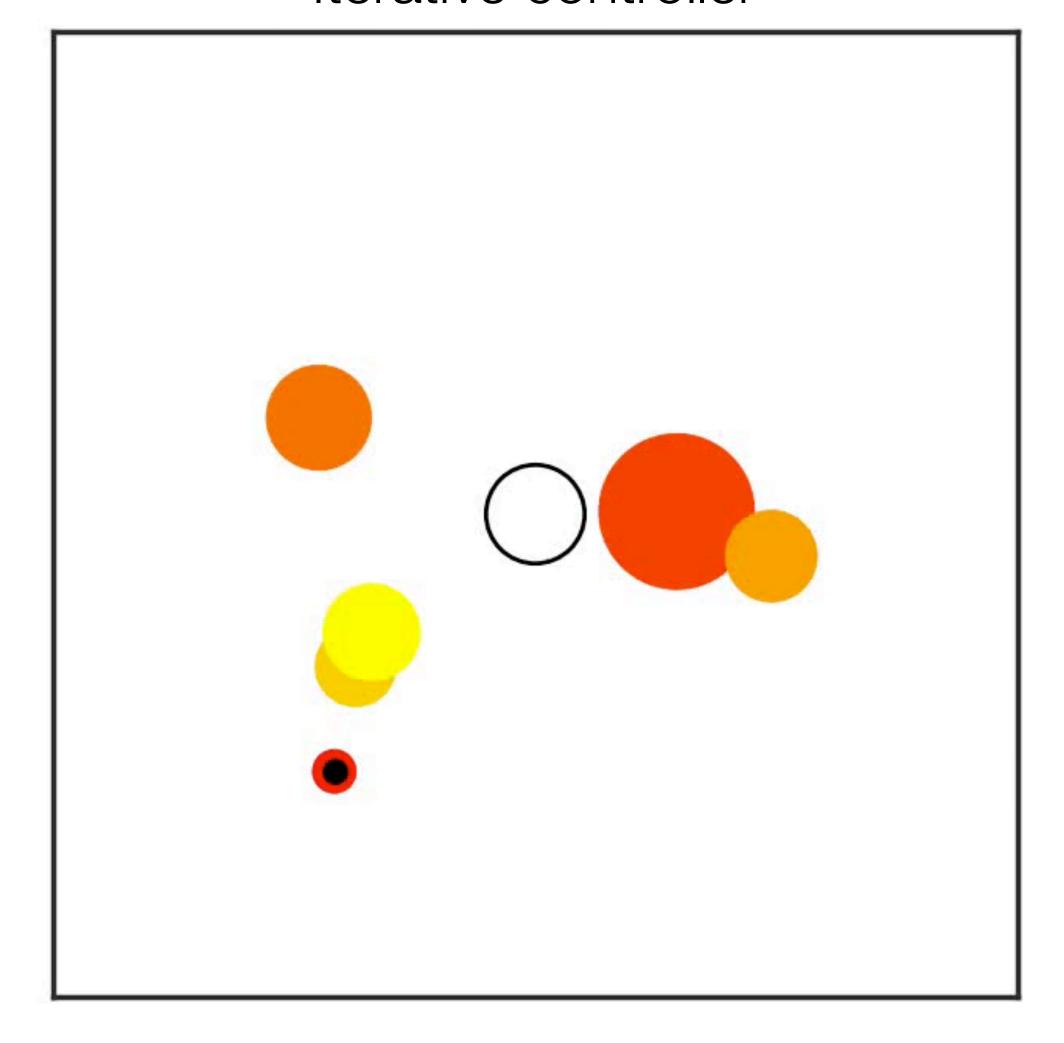
Model of the world that evaluates proposed controls

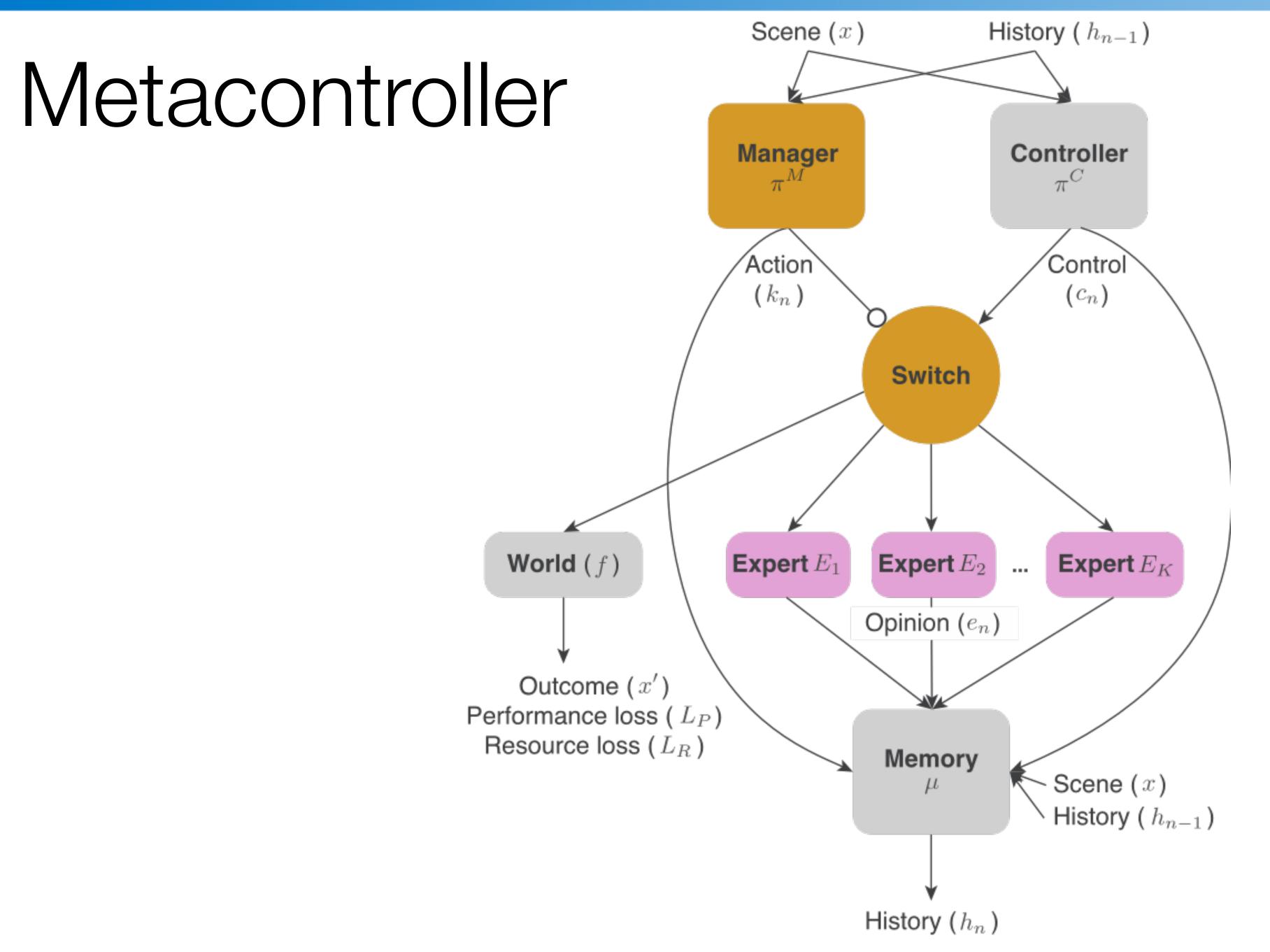
#### Memory (LSTM):







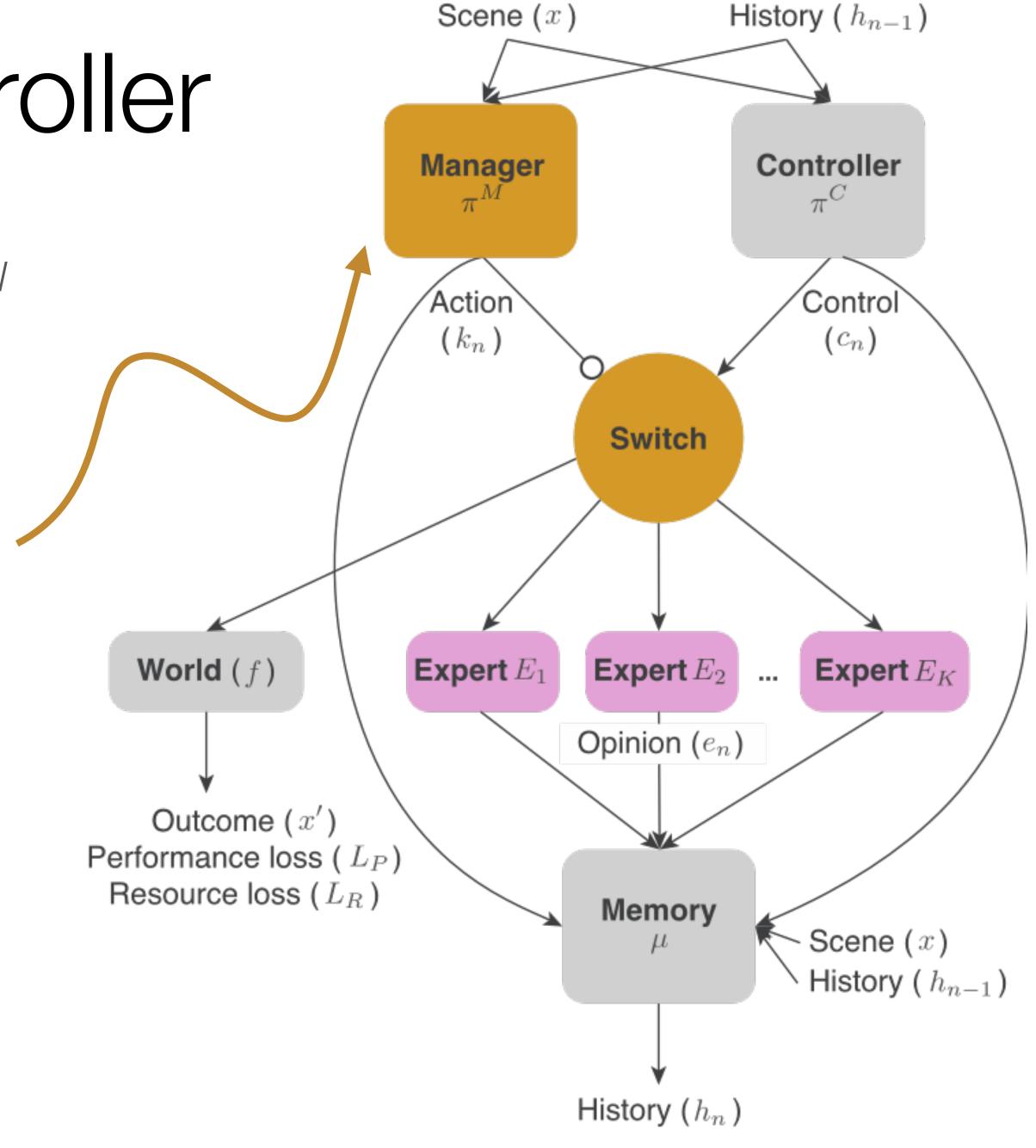








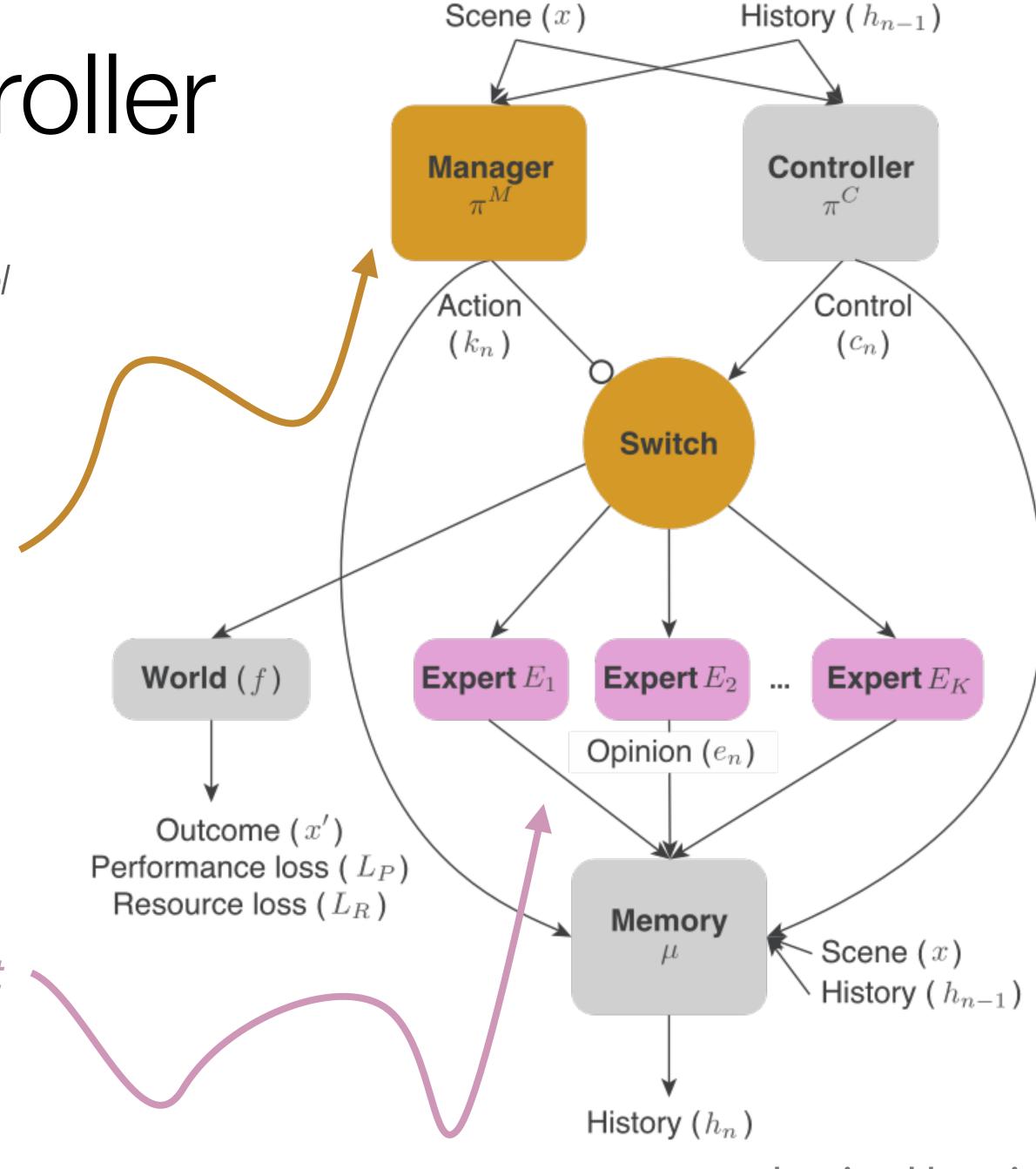
Manager (MLP): Meta-level
policy that determines
whether to send the
proposed control to the
world, or to an expert, in
order to minimize
performance loss +
resource loss





Manager (MLP): Meta-level
policy that determines
whether to send the
proposed control to the
world, or to an expert, in
order to minimize
performance loss +
resource loss

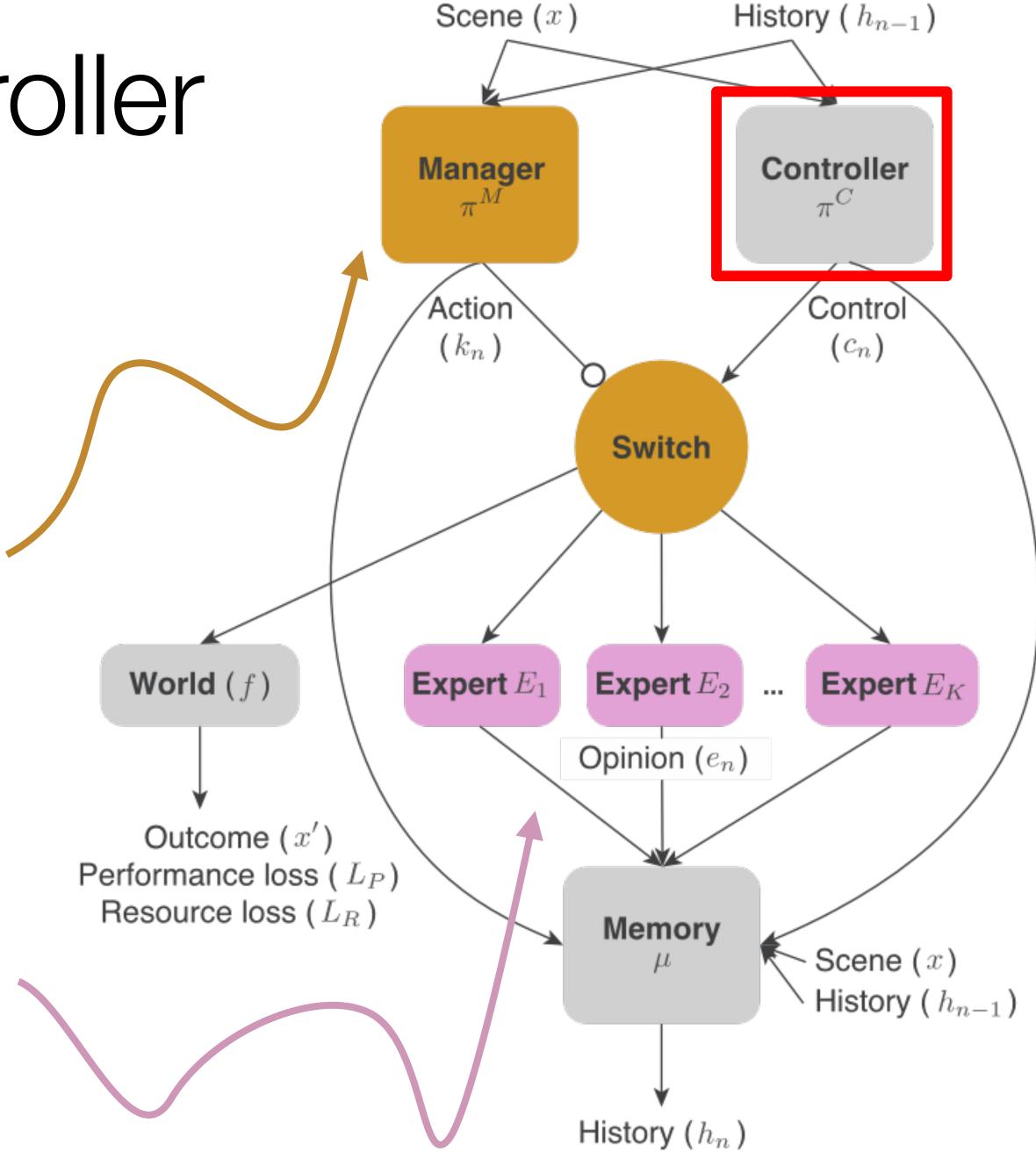
#### Experts:





Manager (MLP): Meta-level
policy that determines
whether to send the
proposed control to the
world, or to an expert, in
order to minimize
performance loss +
resource loss

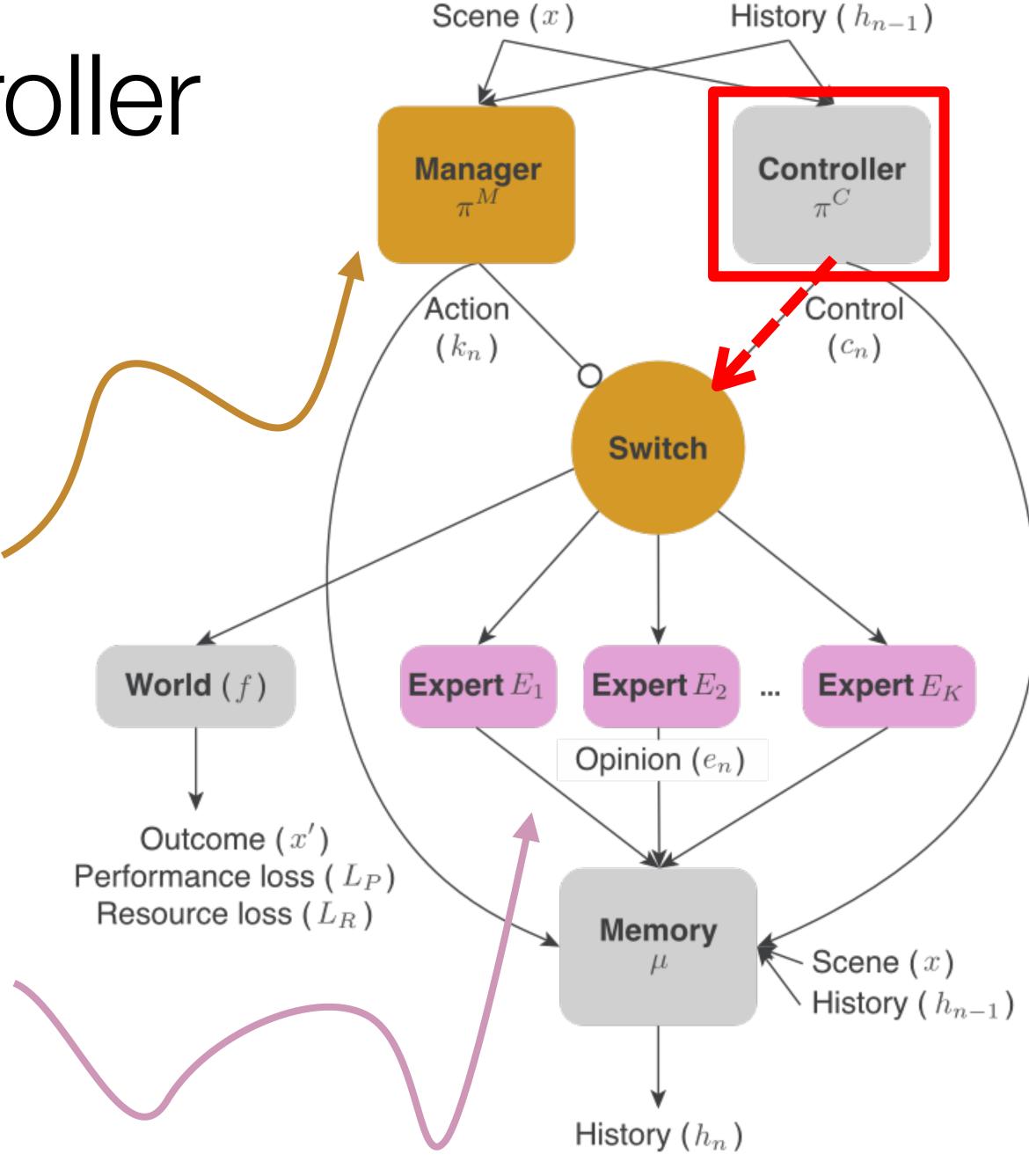
#### Experts:





Manager (MLP): Meta-level
policy that determines
whether to send the
proposed control to the
world, or to an expert, in
order to minimize
performance loss +
resource loss

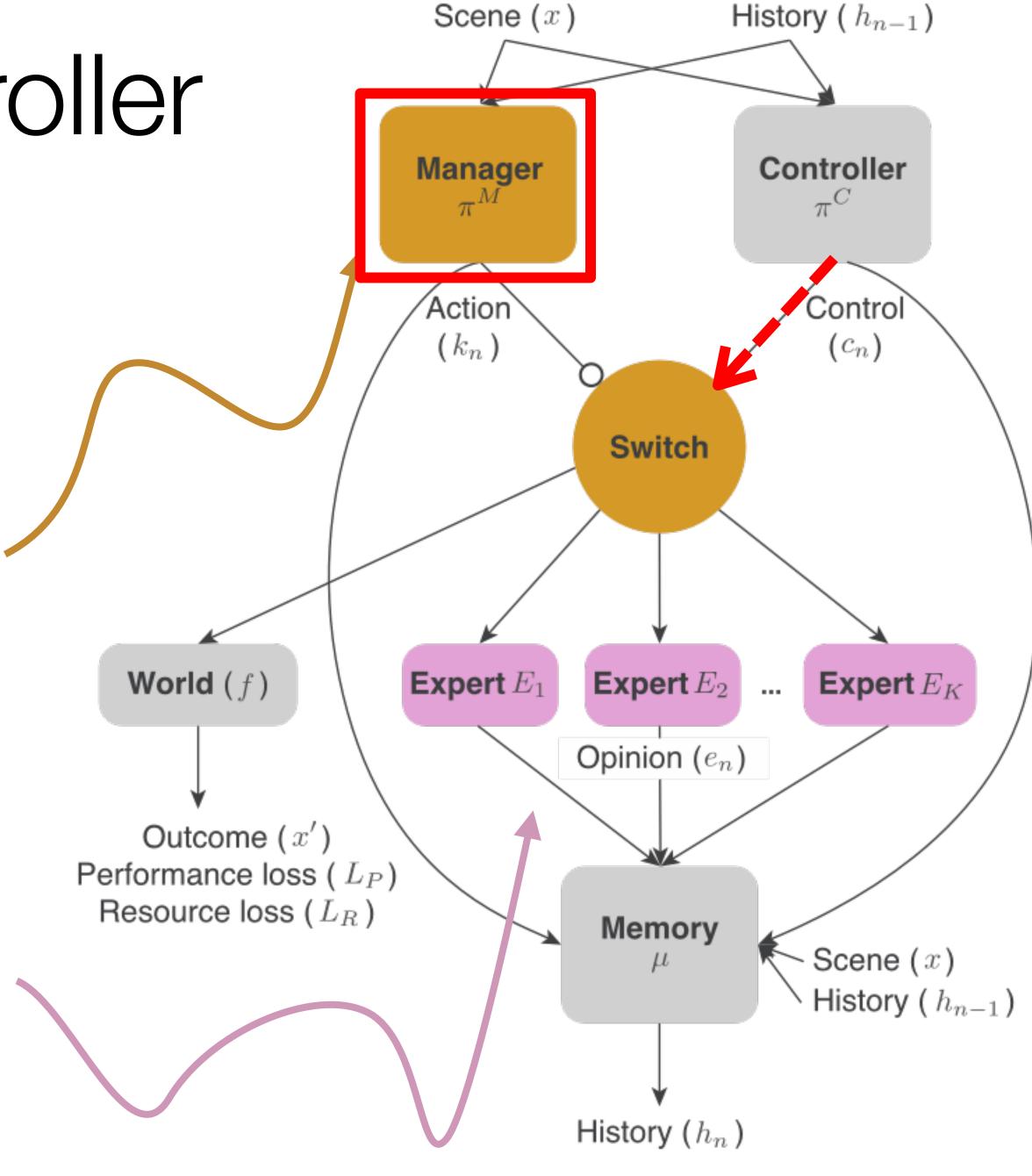
#### Experts:





Manager (MLP): Meta-level
policy that determines
whether to send the
proposed control to the
world, or to an expert, in
order to minimize
performance loss +
resource loss

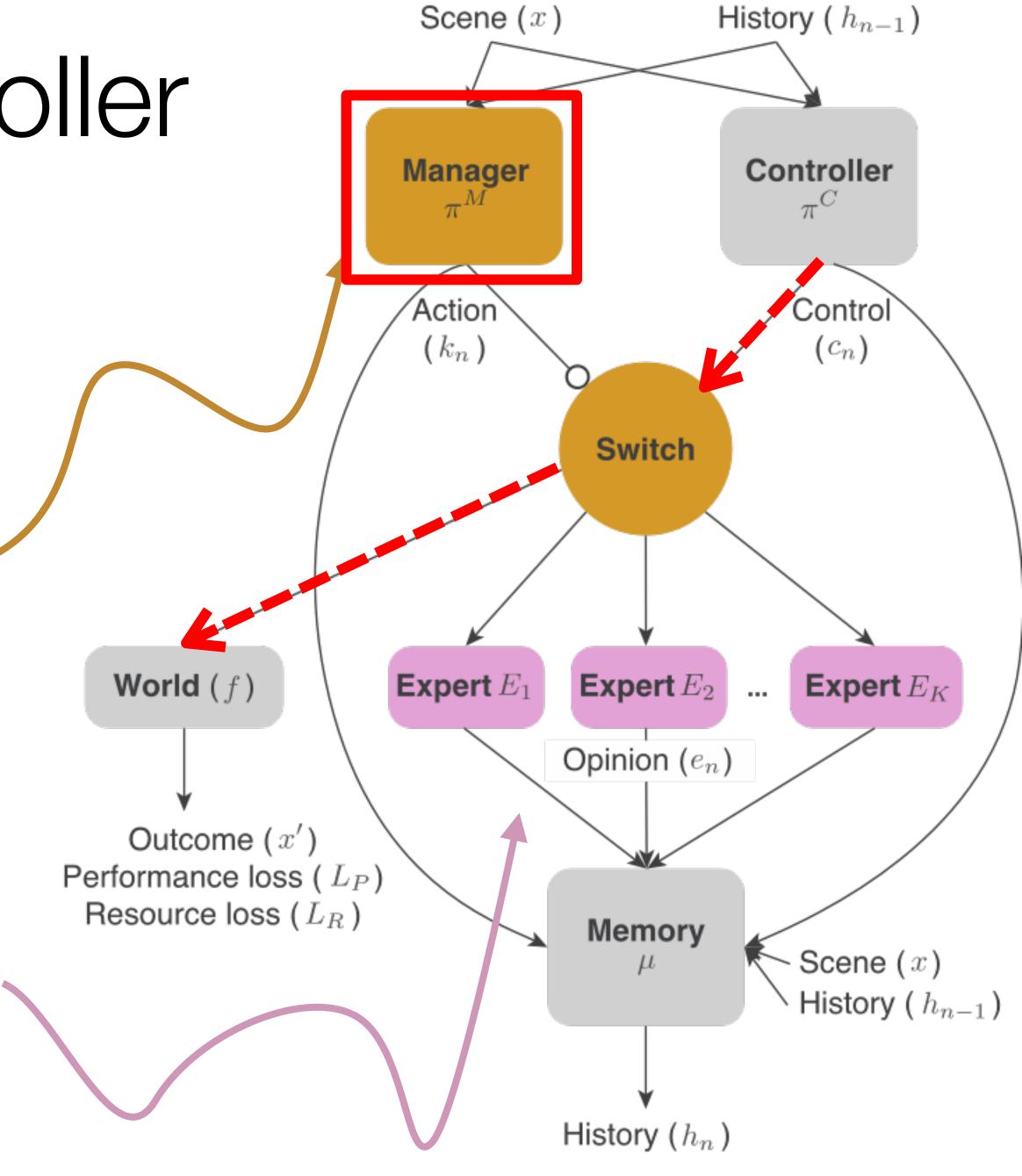
#### Experts:





Manager (MLP): Meta-level
policy that determines
whether to send the
proposed control to the
world, or to an expert, in
order to minimize
performance loss +
resource loss

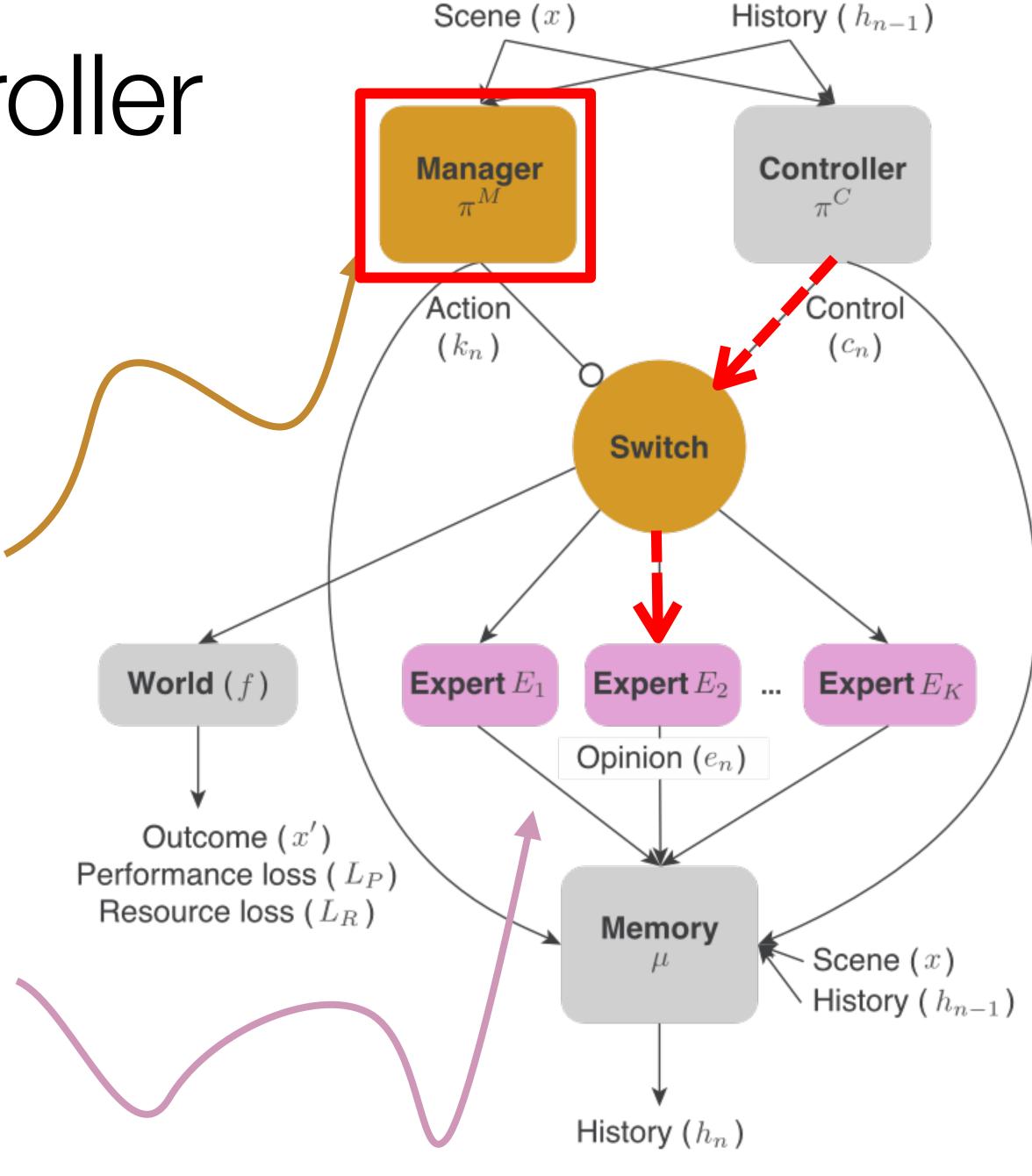
#### Experts:





Manager (MLP): Meta-level
policy that determines
whether to send the
proposed control to the
world, or to an expert, in
order to minimize
performance loss +
resource loss

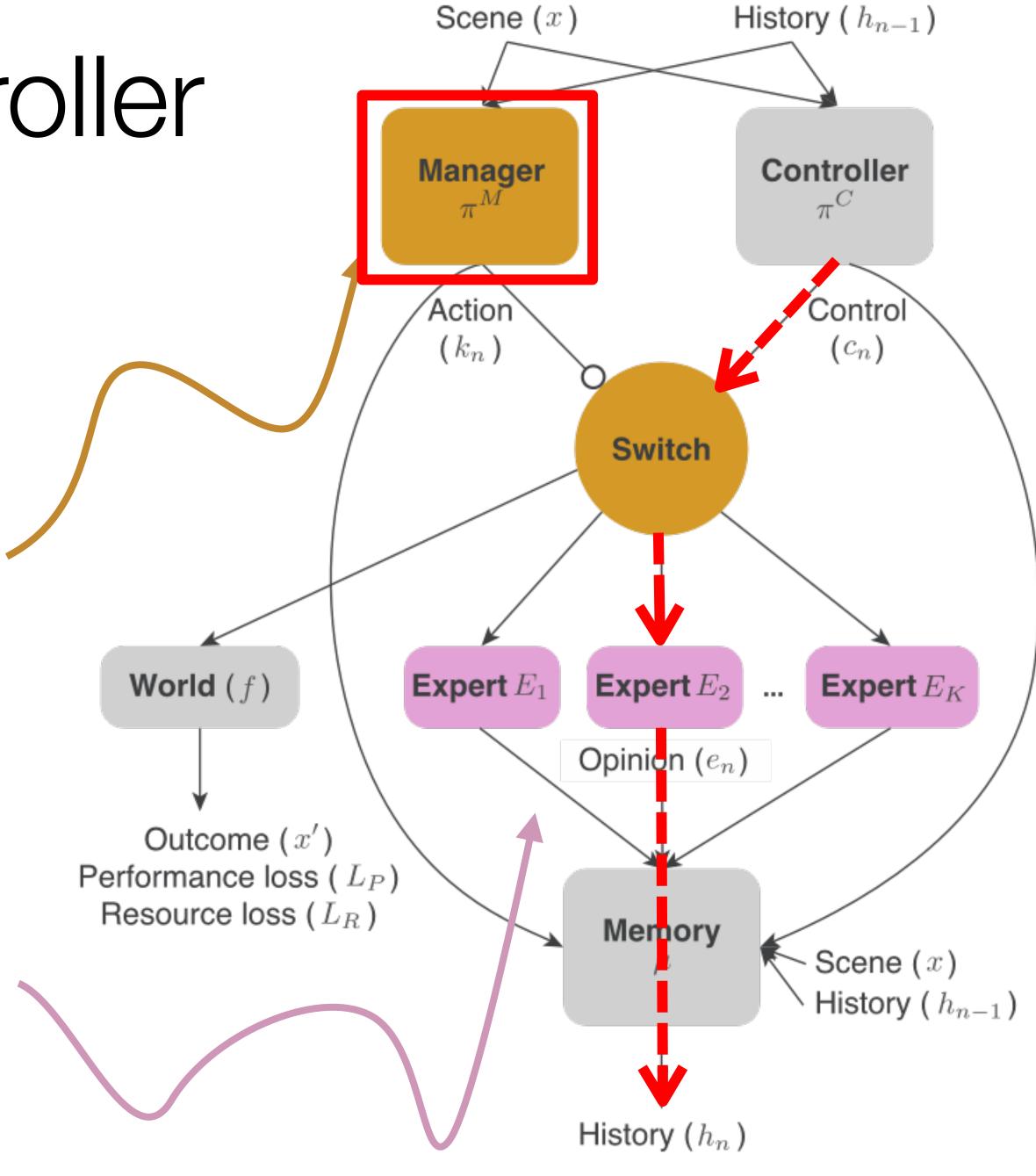
#### Experts:





Manager (MLP): Meta-level
policy that determines
whether to send the
proposed control to the
world, or to an expert, in
order to minimize
performance loss +
resource loss

#### Experts:



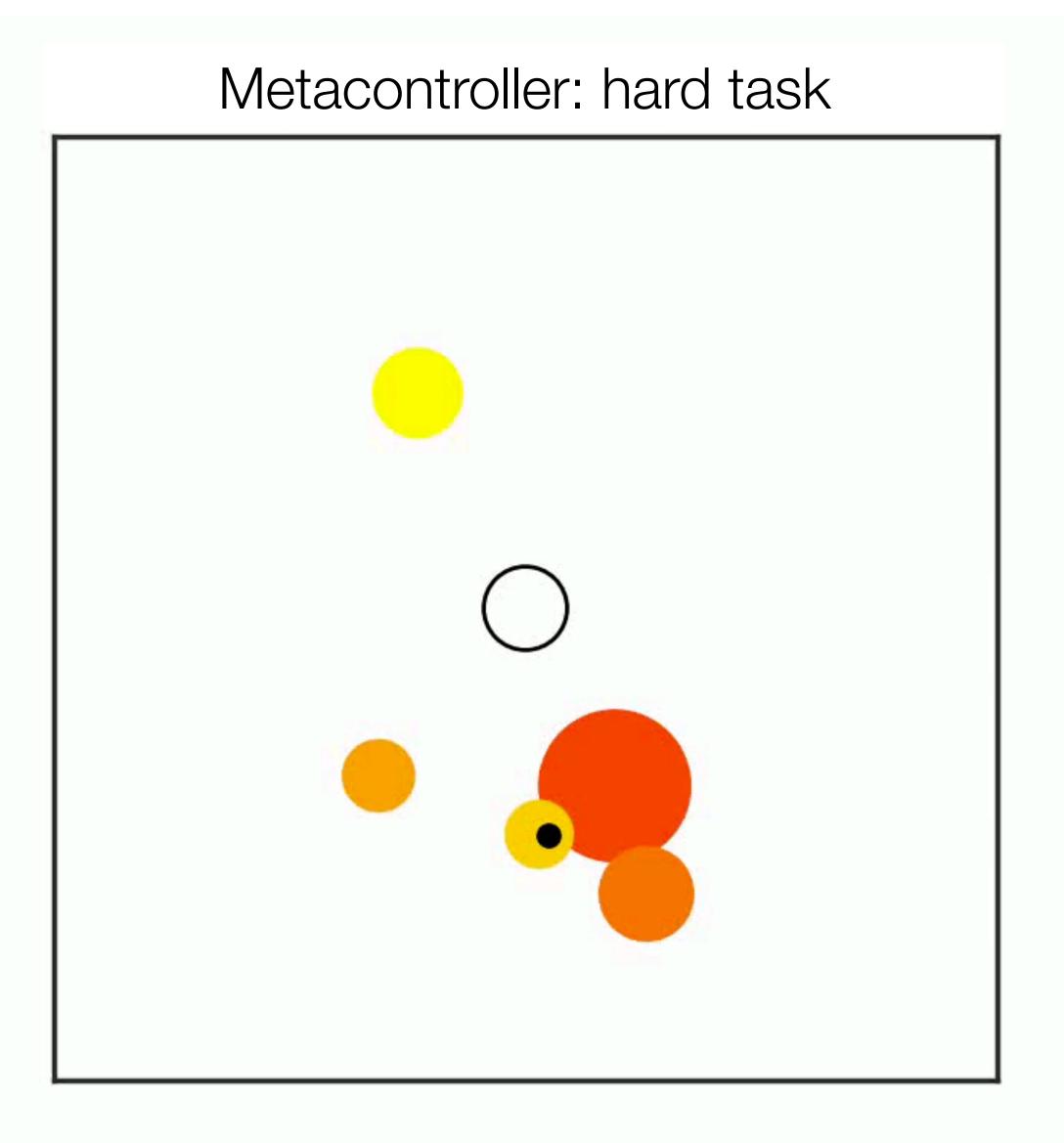


# Metacontroller: simple task

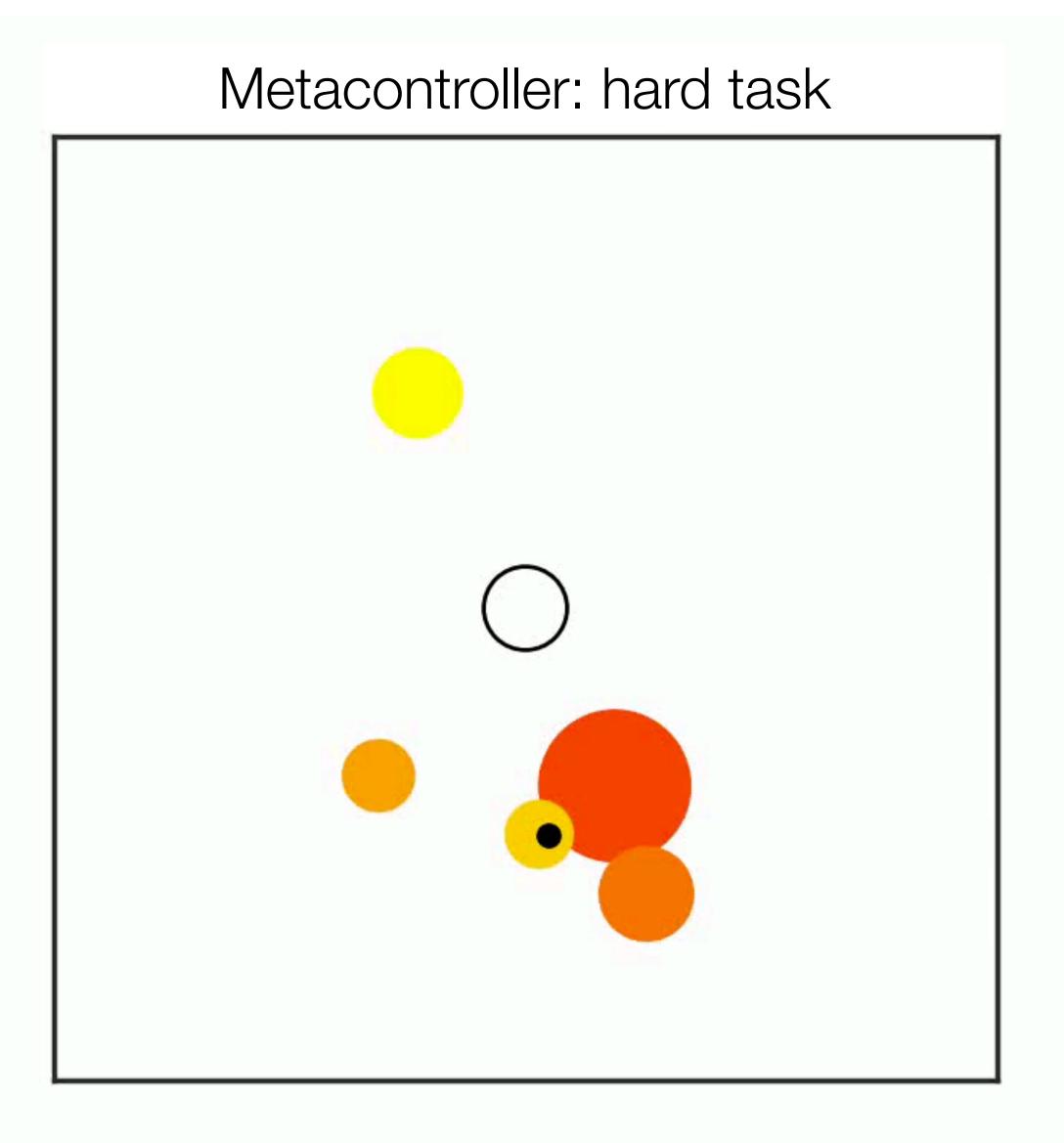


# Metacontroller: simple task





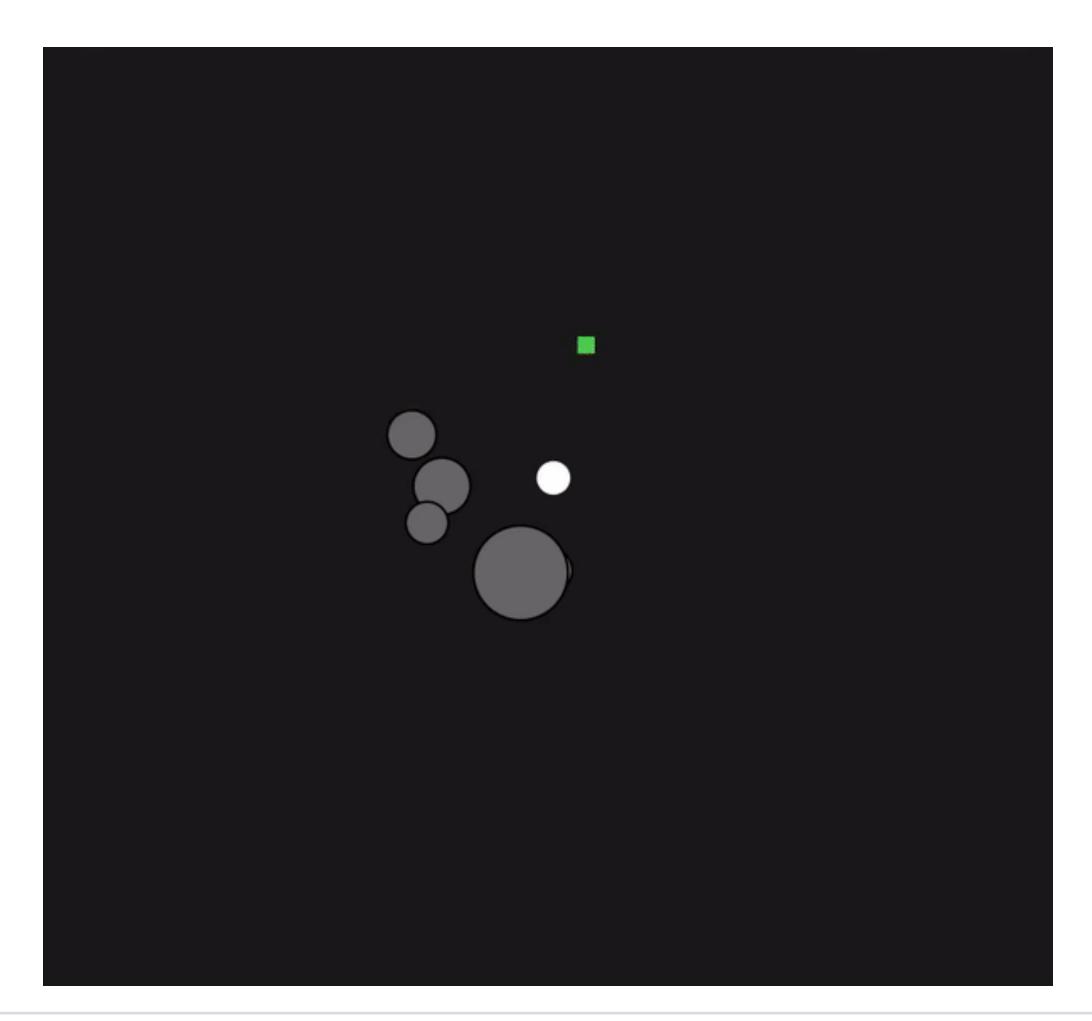


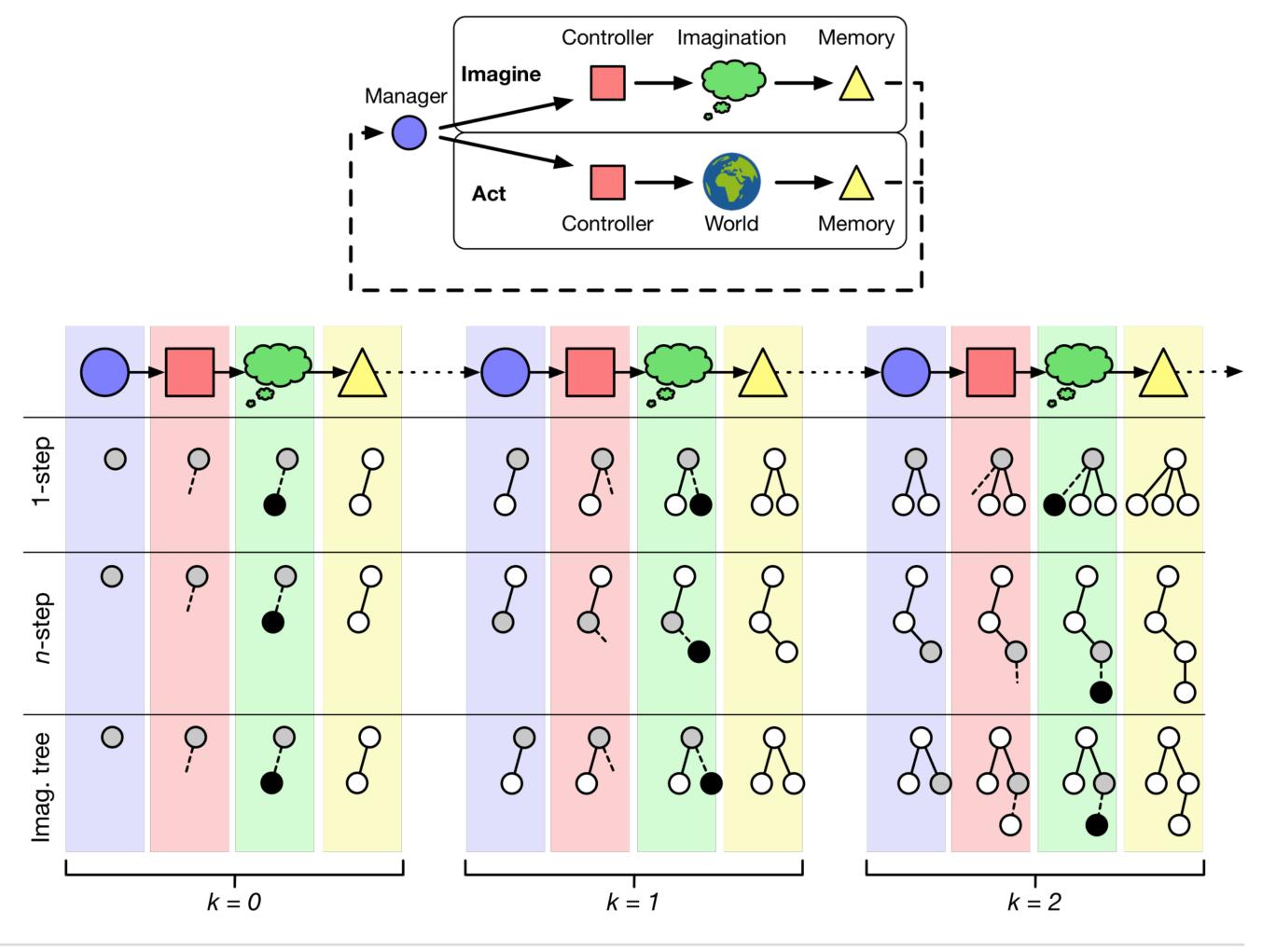




## Imagination-Based Planner

Pascanu, Li, et al. (2017)

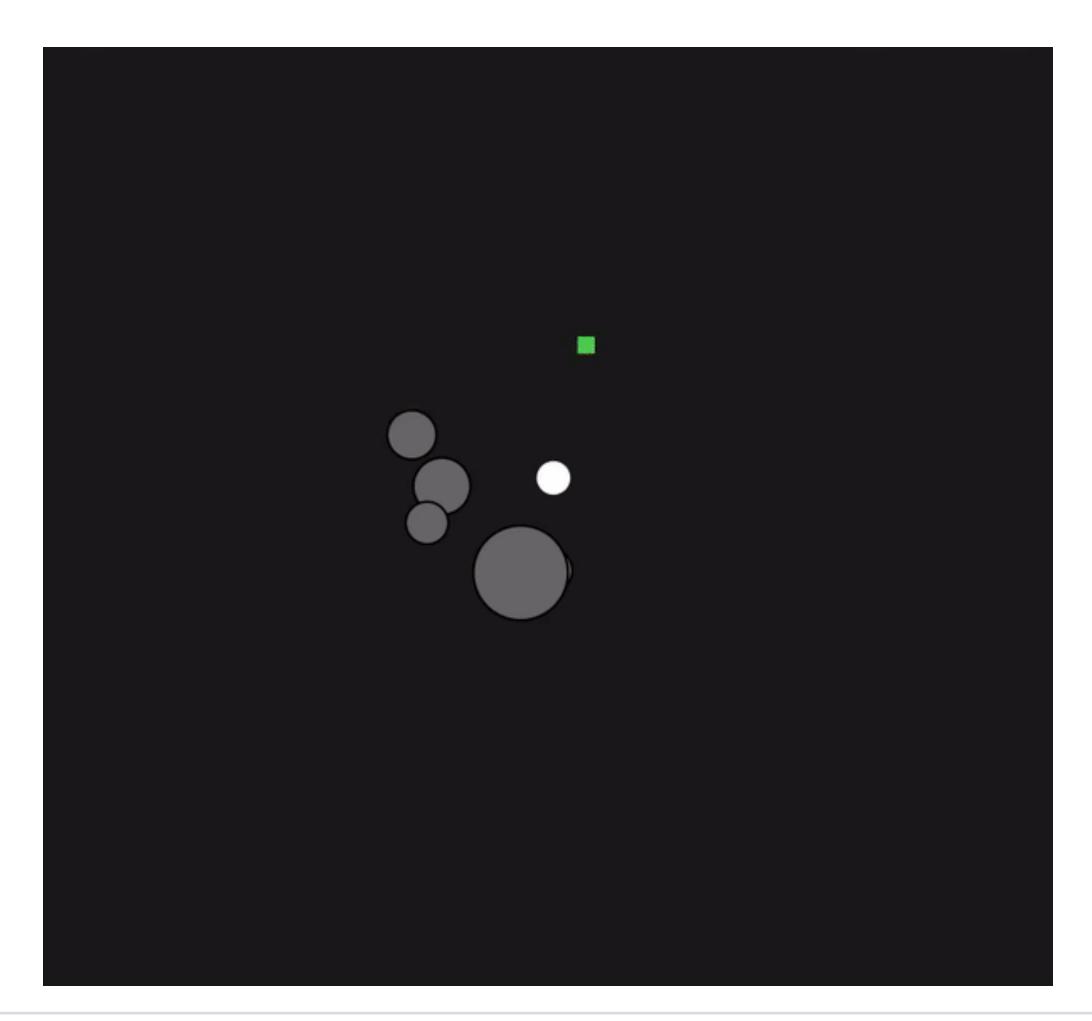


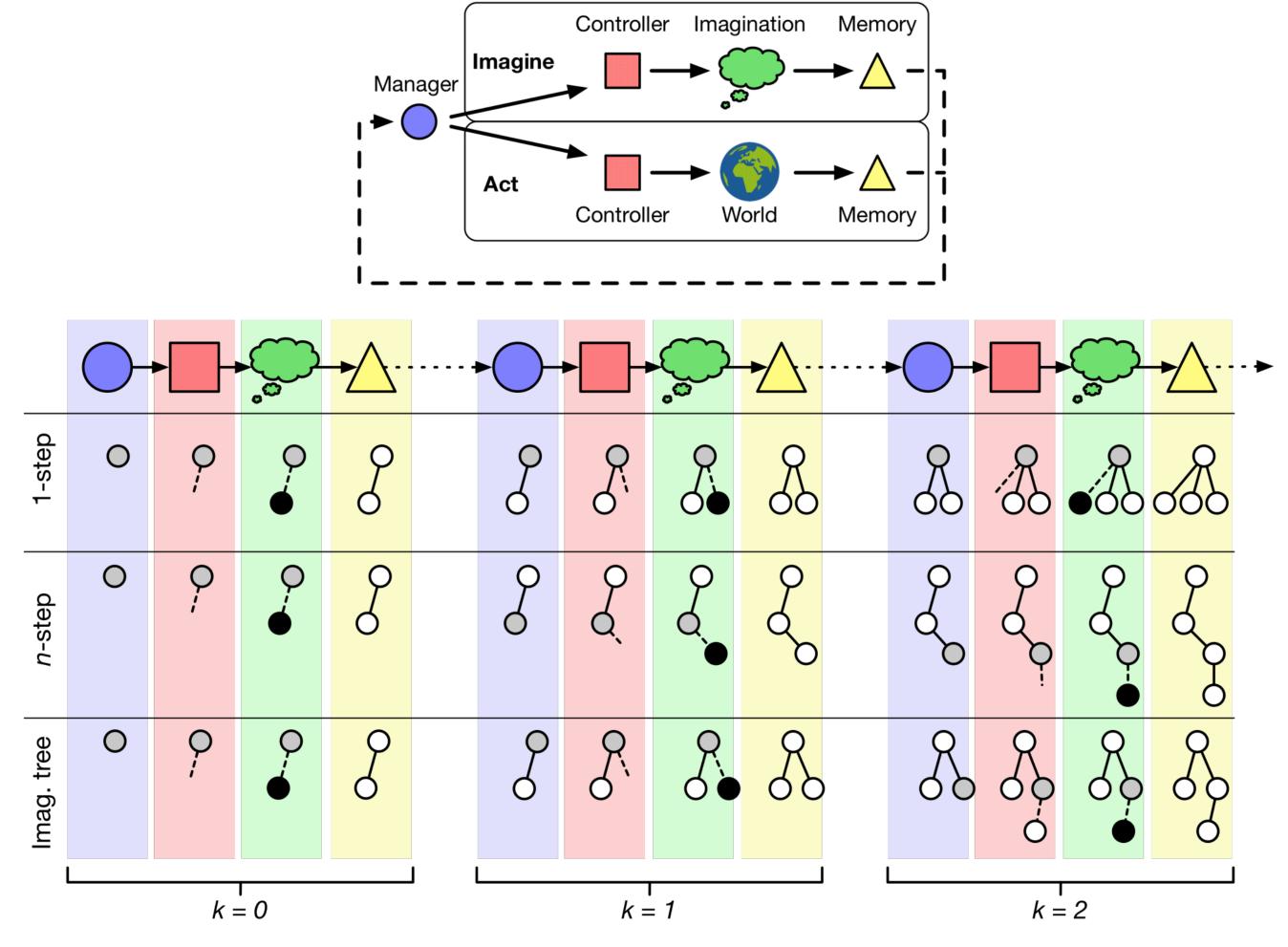




## Imagination-Based Planner

Pascanu, Li, et al. (2017)



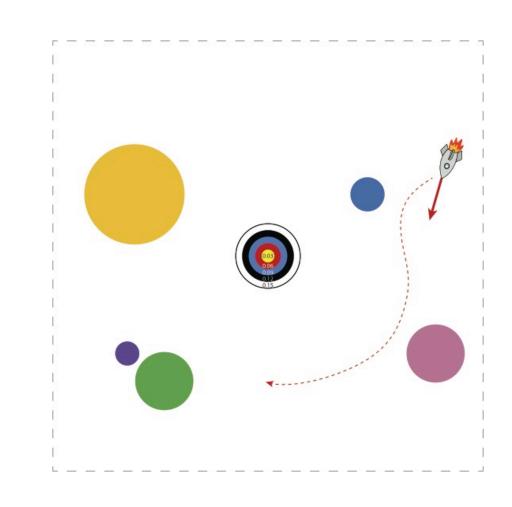




### Outline

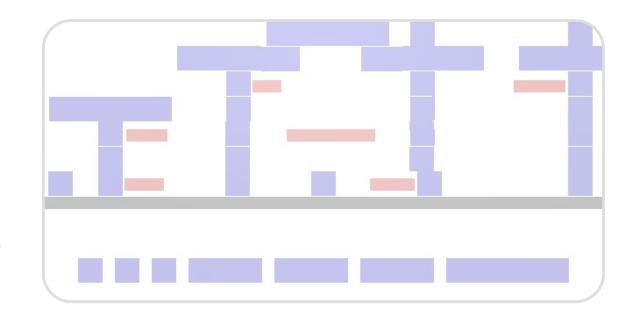
1. **Structured Computation**: the way that individual computations or functions are composed into more complex structures.

Hamrick, Ballard, Pascanu, Vinyals, Heess, & Battaglia (2017, ICLR)



2. Structured Representation: the format of the data that computations are performed over.

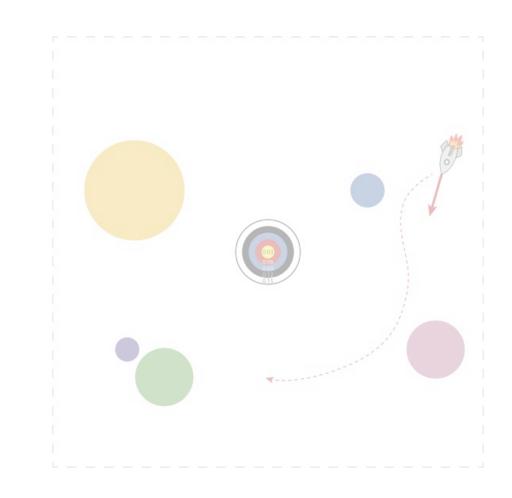
Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018, arXiv) Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia, & Hamrick (2019, ICML)



#### Outline

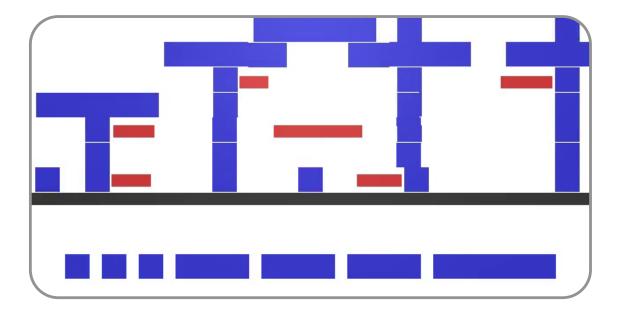
1. Structured Computation: the way that individual computations or functions are composed into more complex structures.

Hamrick, Ballard, Pascanu, Vinyals, Heess, & Battaglia (2017, ICLR)



2. Structured Representation: the format of the data that computations are performed over.

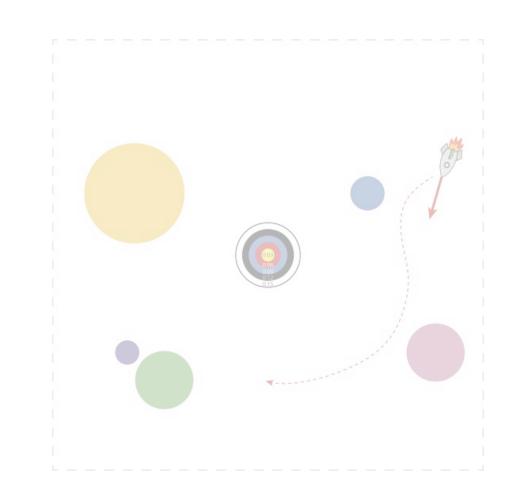
Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018, arXiv) Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia, & Hamrick (2019, ICML)



### Outline

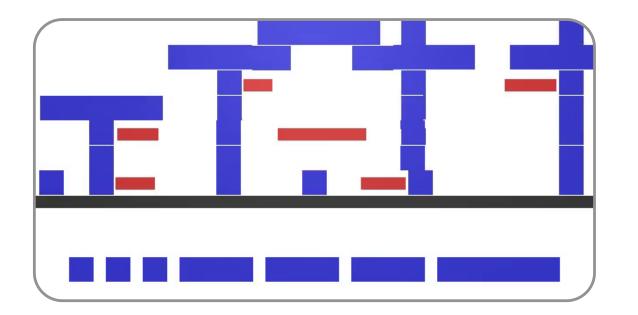
1. Structured Computation: the way that individual computations or functions are composed into more complex structures.

Hamrick, Ballard, Pascanu, Vinyals, Heess, & Battaglia (2017, ICLR)



2. Structured Representation: the format of the data that computations are performed over.

Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018, arXiv) Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia, & Hamrick (2019, ICML)

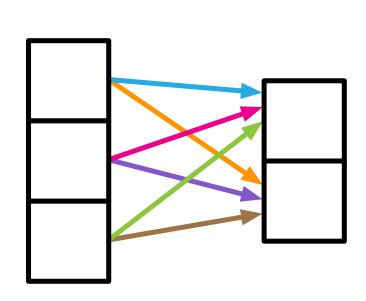




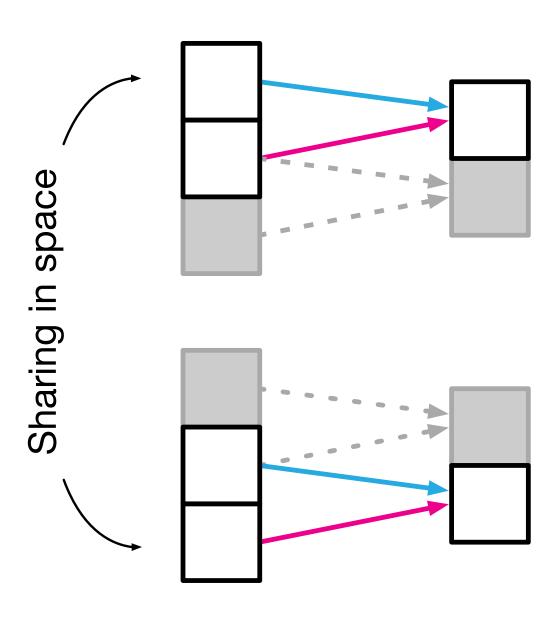
Led by Victor Bapst and Alvaro Sanchez-Gonzalez



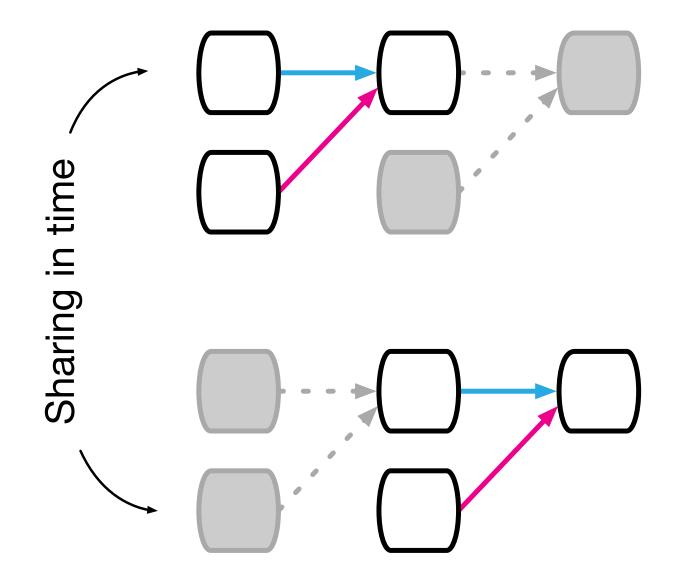
## Structure in Deep Networks



Fully-Connected Layer
Unstructured computation
Unstructured representation



Convolutional Layer
Structured computation
Unstructured representation



Recurrent Layer

Structured computation

Unstructured representation





Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017)



Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017) Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018)



Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017) Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018)

1. Take graphs as input, return graphs as output



Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017) Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018)

1. Take graphs as input, return graphs as output

2. Invariant to the permutation of the nodes and edges

Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017) Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018)

1. Take graphs as input, return graphs as output

2. Invariant to the permutation of the nodes and edges

3. Scale to different numbers of nodes and edges

Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017) Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018)



Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017) Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018)

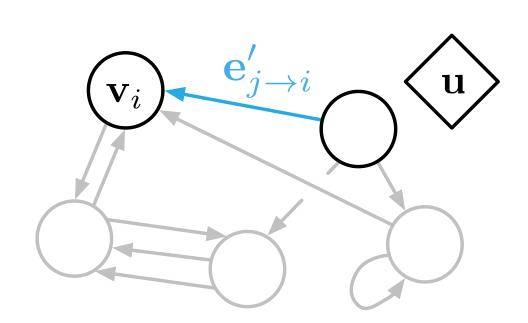
Edges	Nodes	Globals
E	V	$\mathbf{u}$



Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017) Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018)

Edges	Nodes	Globals
E	V	$\mathbf{u}$

#### Edge update

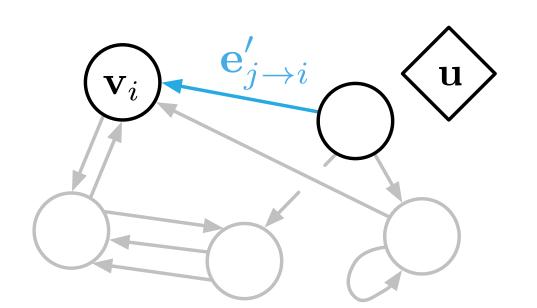


$$\mathbf{e}'_{i\to j} = \phi_e(\mathbf{v}_i, \mathbf{v}_j, \mathbf{e}_{i\to j}, \mathbf{u})$$

Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017) Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018)

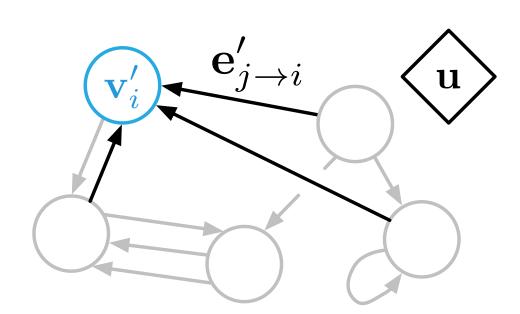
Edges	Nodes	Globals
$\boldsymbol{E}$	V	$\mathbf{u}$

#### Edge update



$$\mathbf{e}'_{i \to j} = \phi_e(\mathbf{v}_i, \mathbf{v}_j, \mathbf{e}_{i \to j}, \mathbf{u})$$

#### Node update

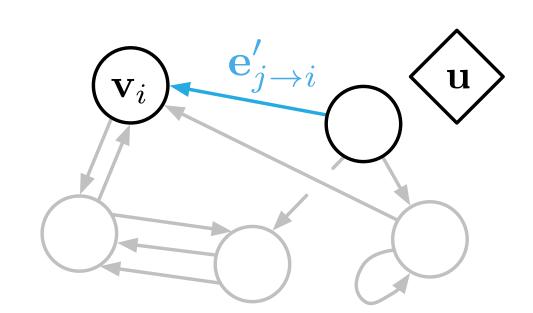


$$\mathbf{e}'_{i\to j} = \phi_e(\mathbf{v}_i, \mathbf{v}_j, \mathbf{e}_{i\to j}, \mathbf{u})$$
  $\mathbf{v}'_i = \phi_v(\mathbf{v}_i, \sum_j \mathbf{e}'_{j\to i}, \mathbf{u})$ 

Gori et al. (2005), Scarselli et al. (2005), Scarselli et al. (2009), Li et al. (2015), Gilmer et al. (2017) Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018)

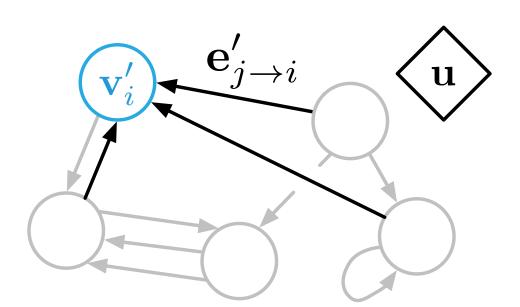
Edges	Nodes	Globals
$\boldsymbol{E}$	V	$\mathbf{u}$

#### Edge update



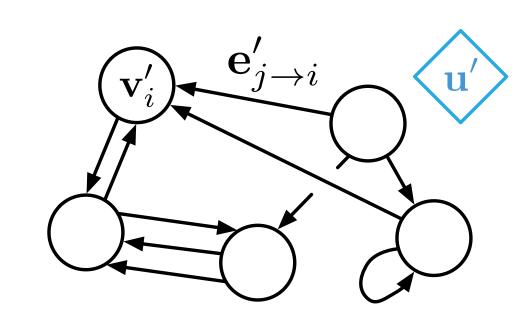
$$\mathbf{e}'_{i\to j} = \phi_e(\mathbf{v}_i, \mathbf{v}_j, \mathbf{e}_{i\to j}, \mathbf{u})$$

#### Node update



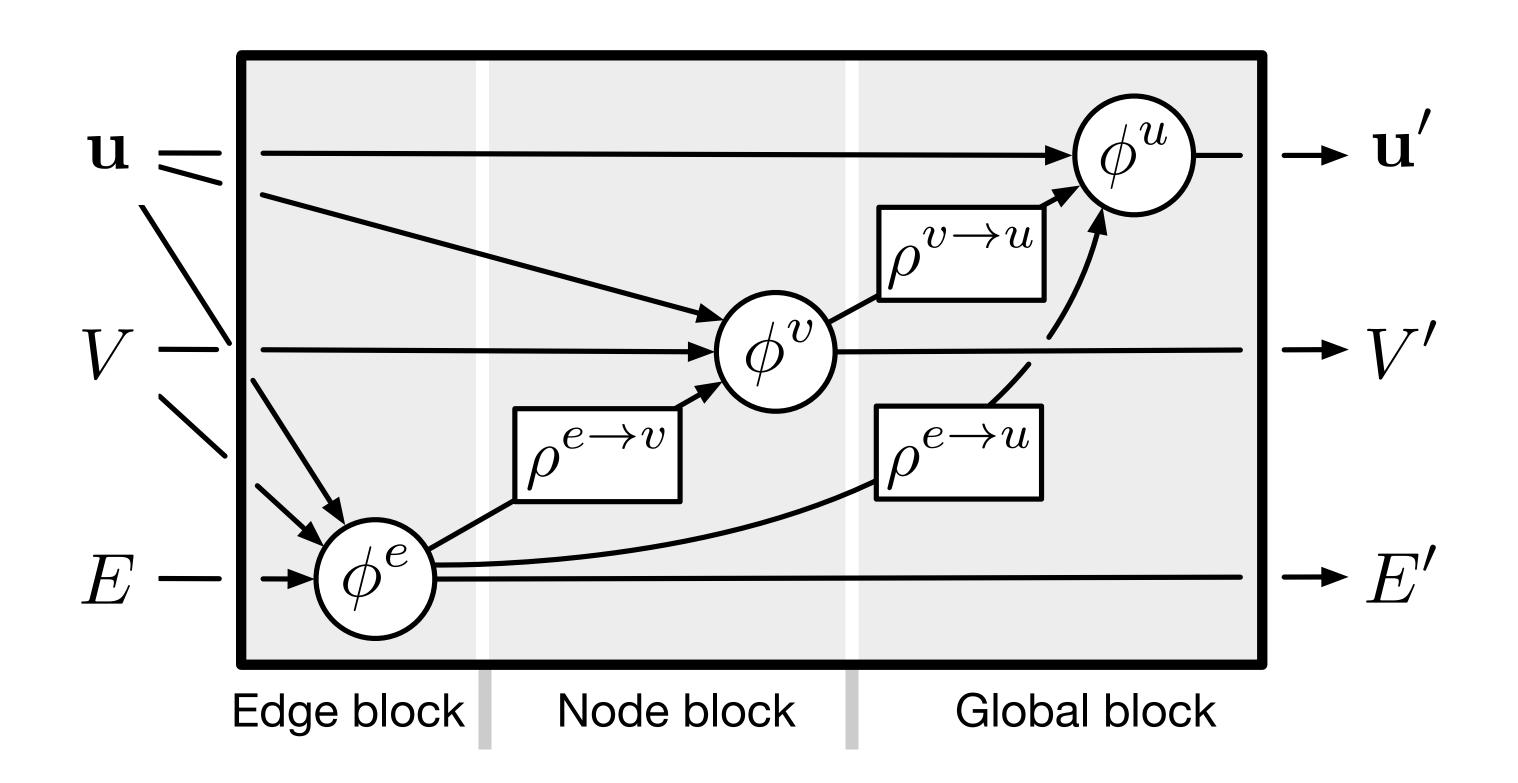
$$\mathbf{v}_i' = \phi_v(\mathbf{v}_i, \sum_j \mathbf{e}_{j \to i}', \mathbf{u})$$

#### Globals update

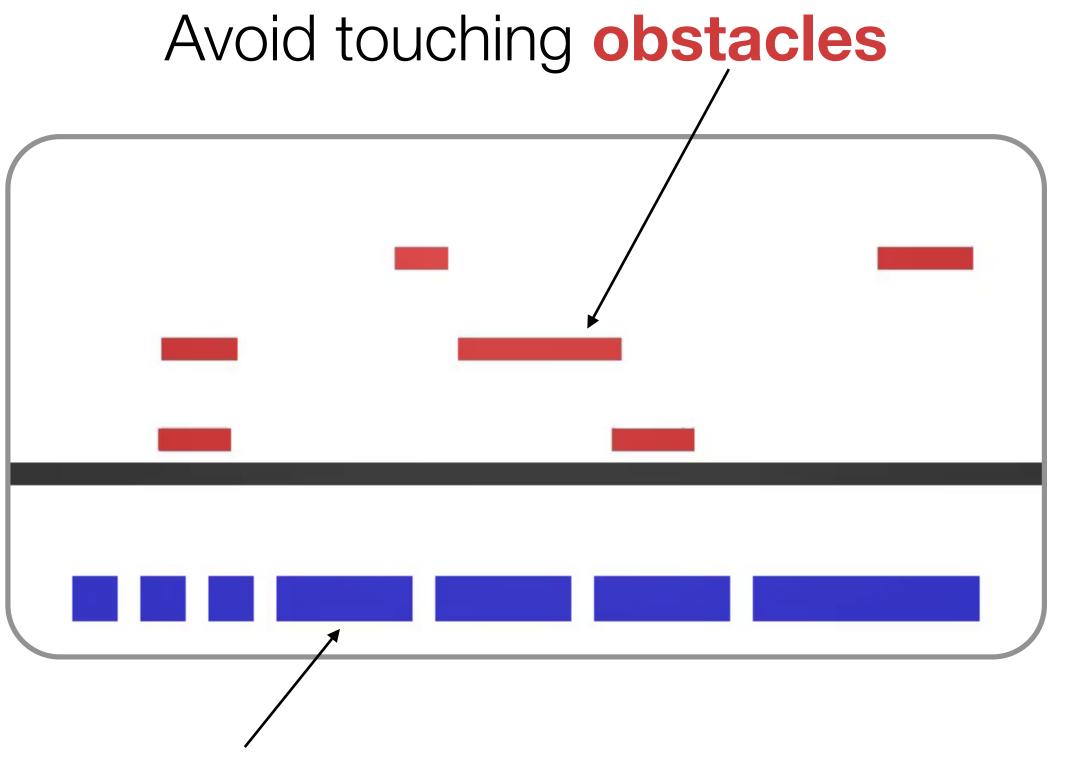


$$\mathbf{u}' = \phi_u(\sum_i \mathbf{v}'_i, \sum_{i,j} \mathbf{e}'_{i \to j}, \mathbf{u})$$

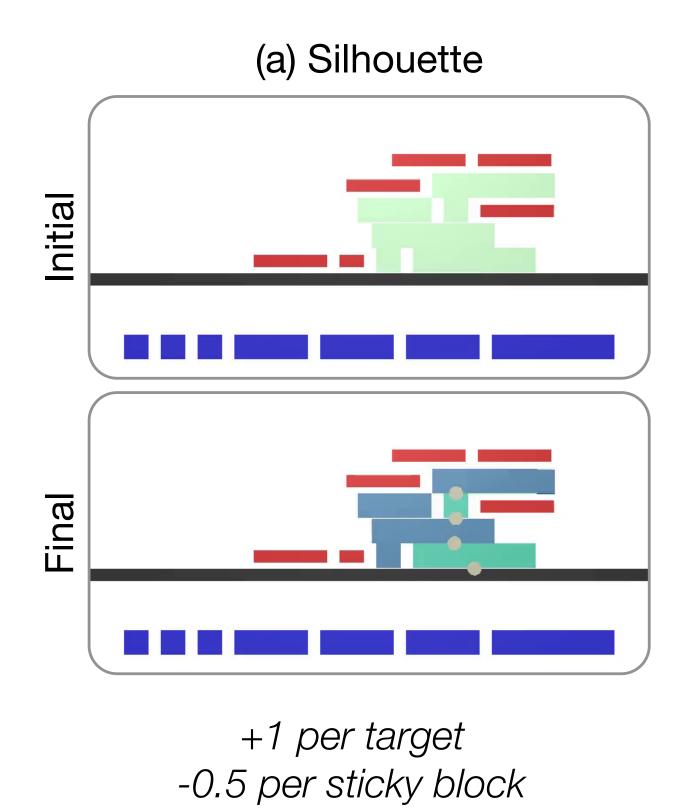
Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (arXiv 2018)

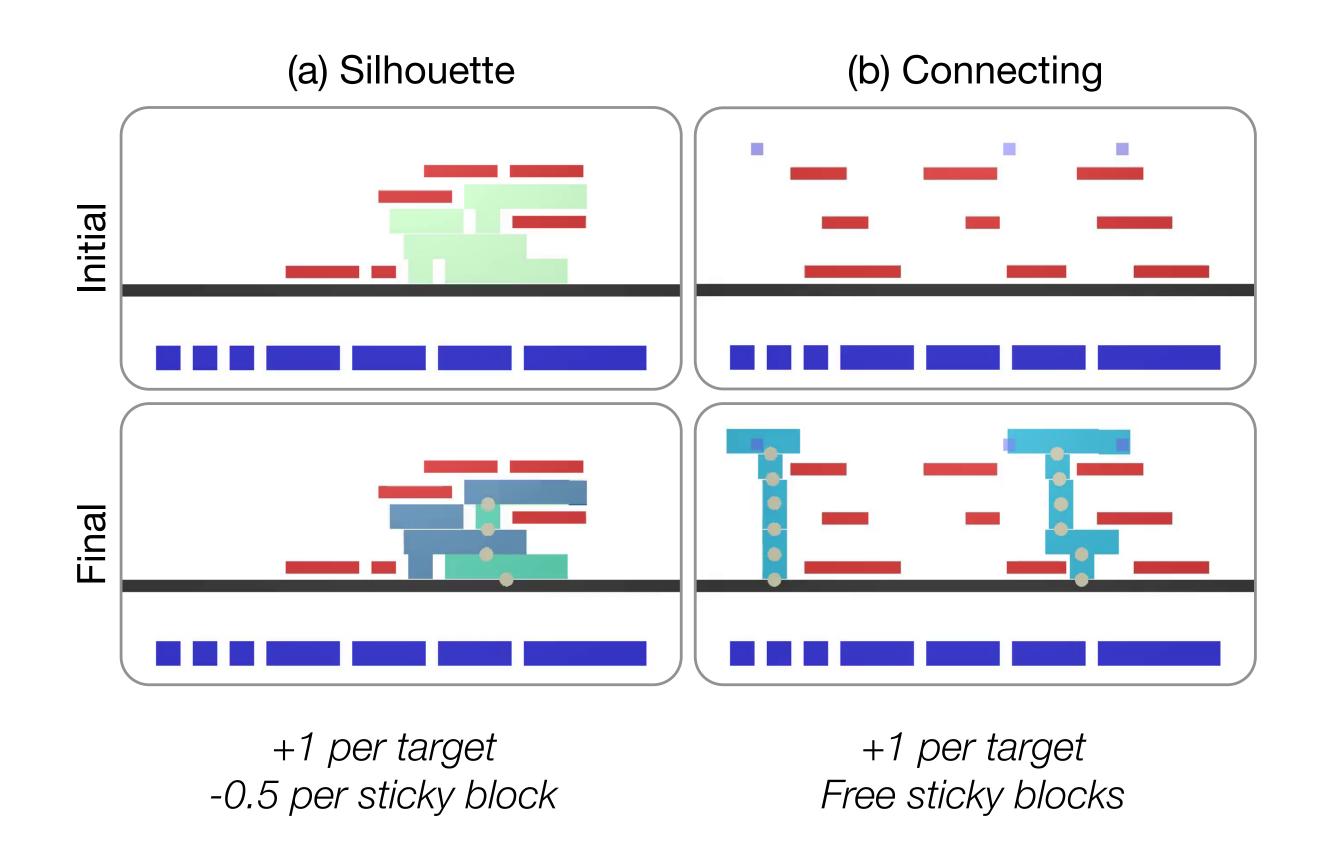


Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

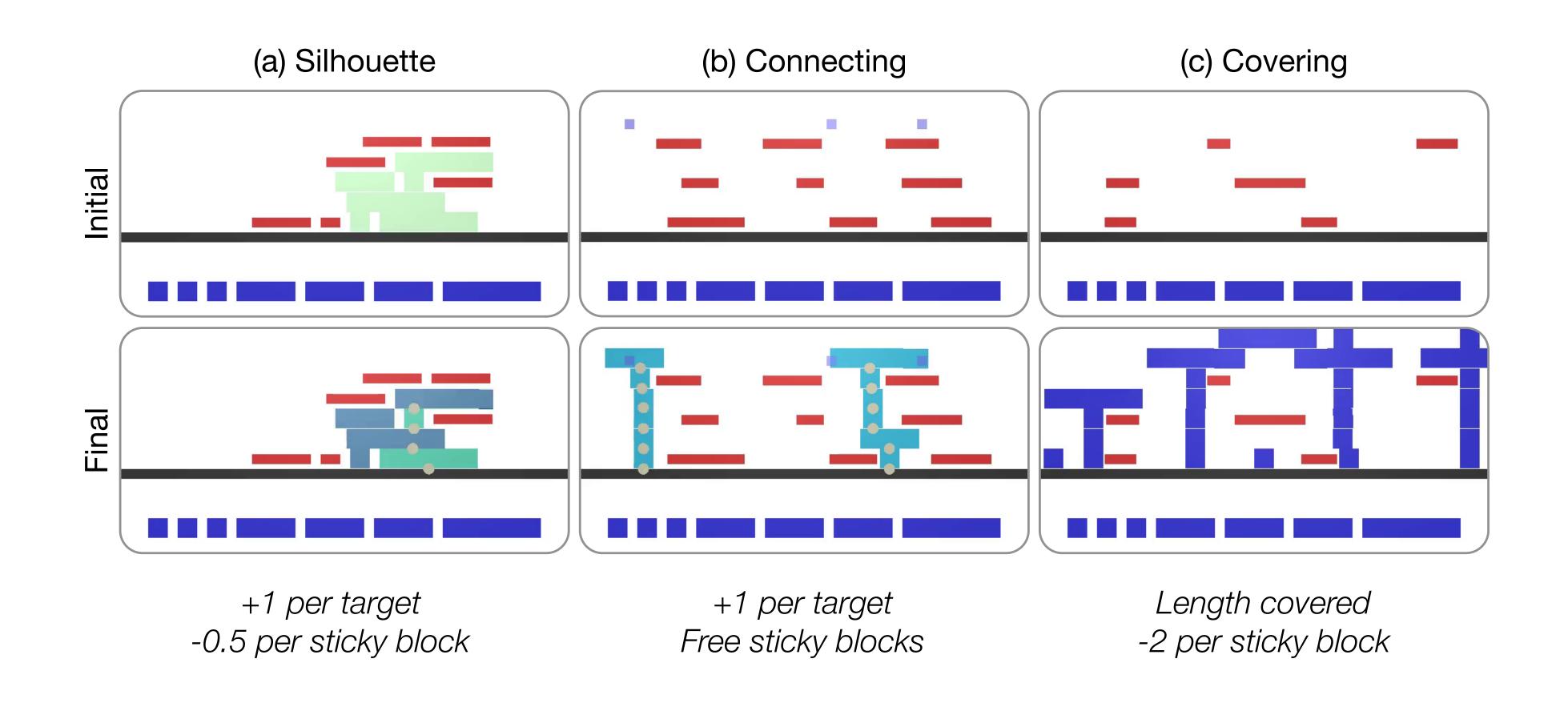


Pick up **blocks** and place them in the scene (and optionally make them **sticky**)

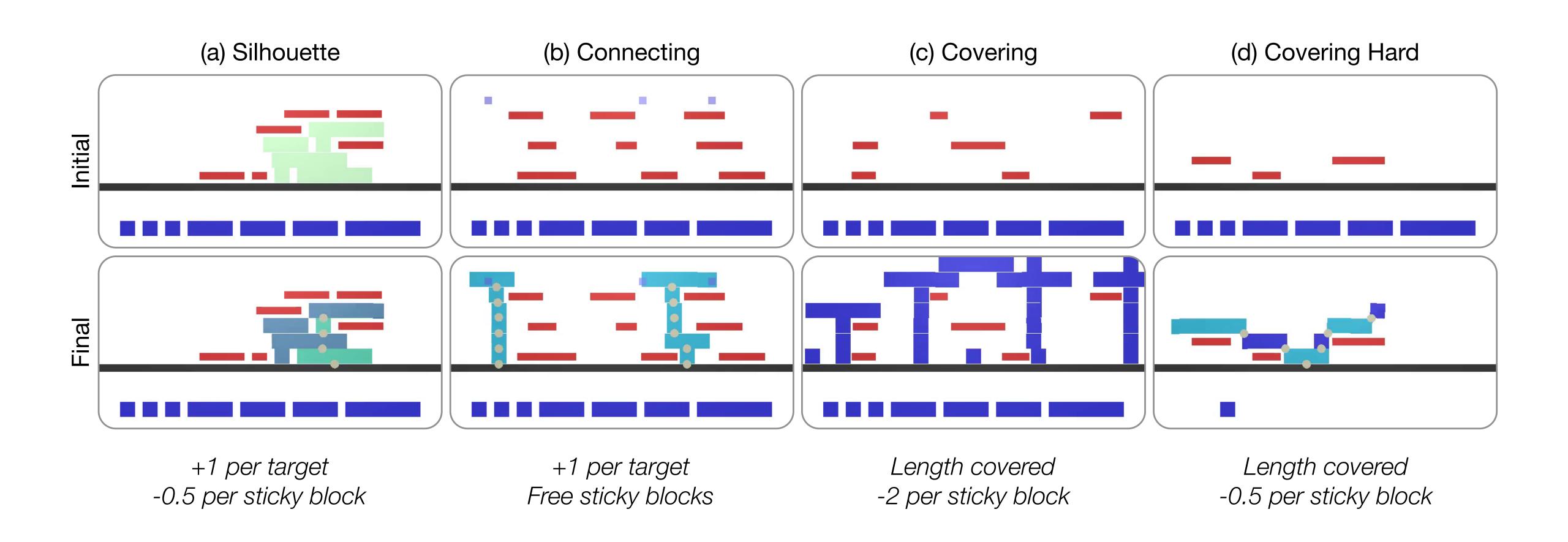






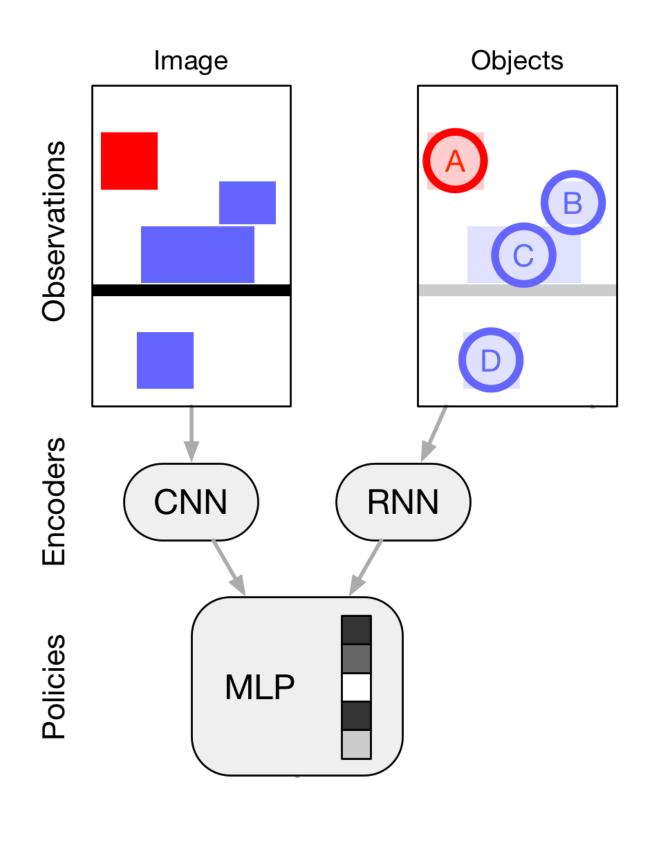








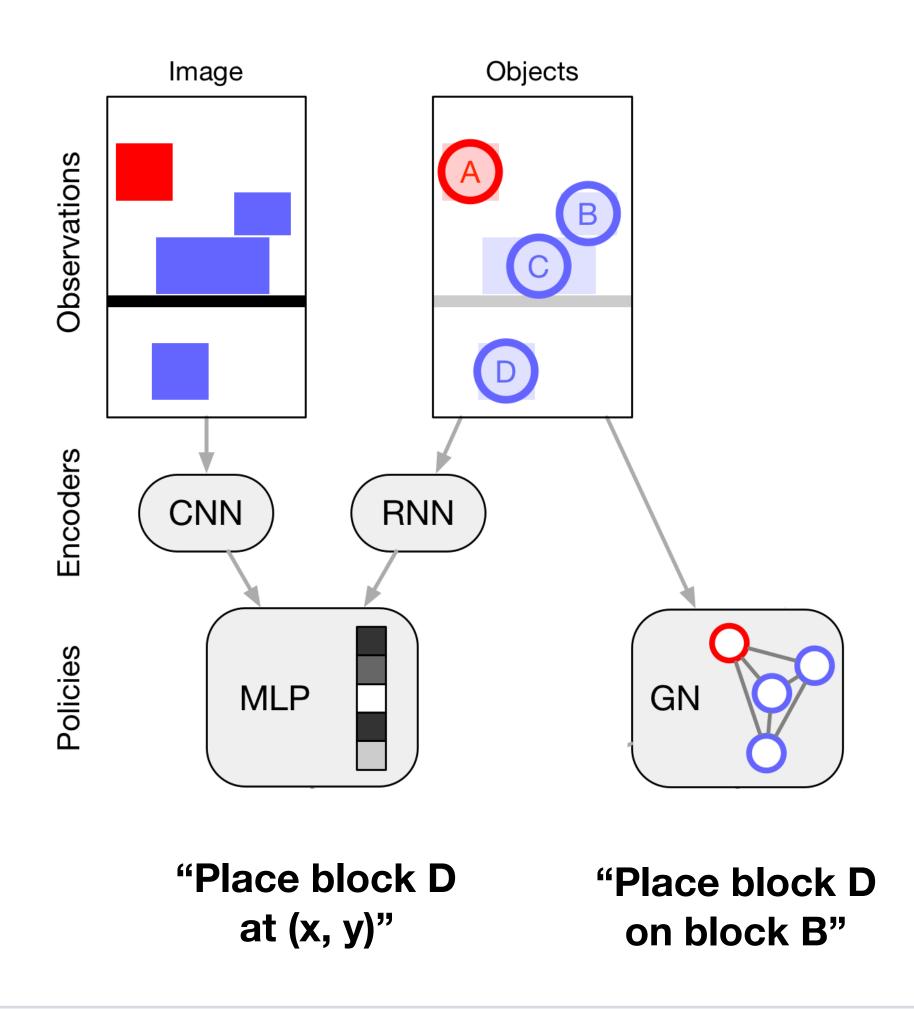
### Absolute and Relative Actions



"Place block D at (x, y)"

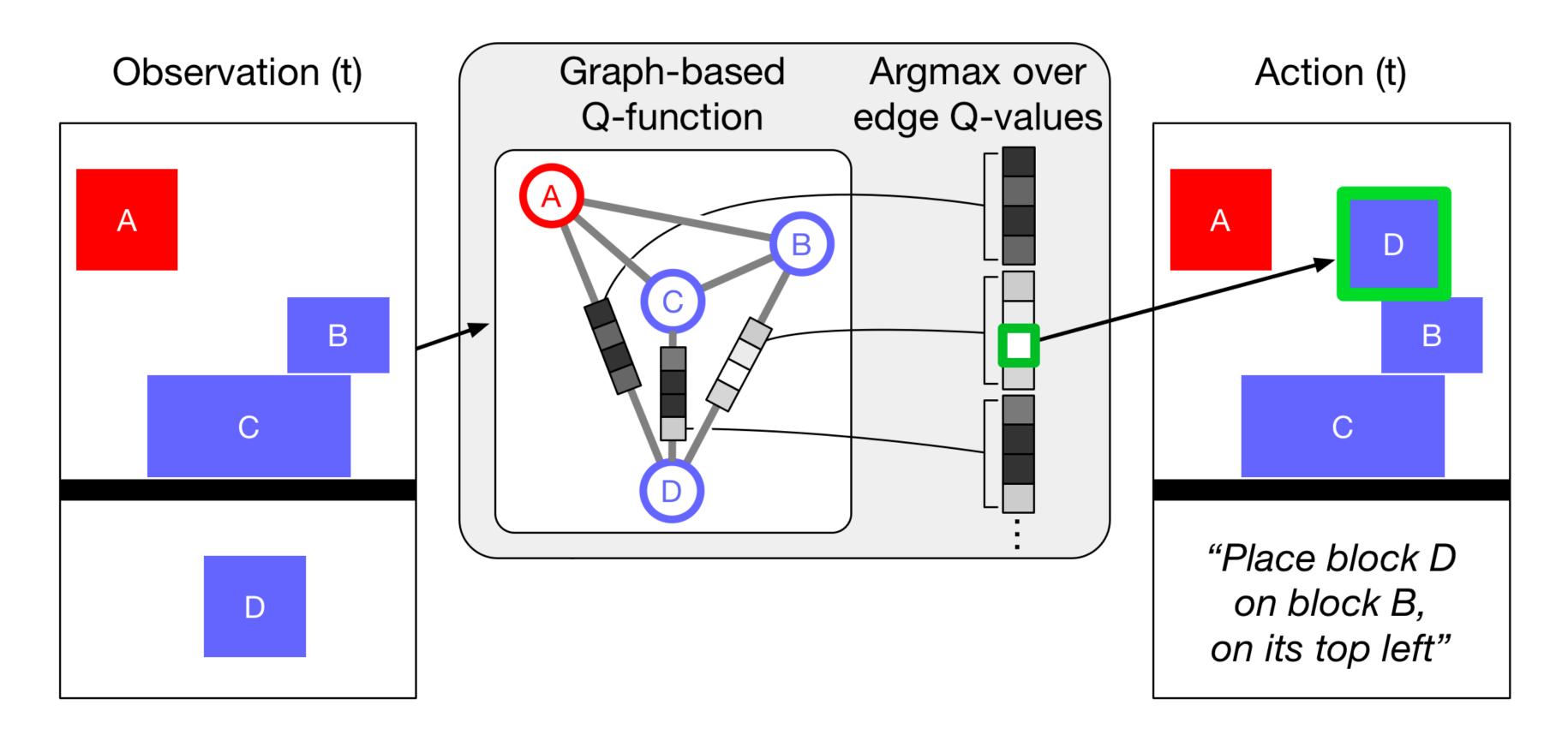


#### Absolute and Relative Actions



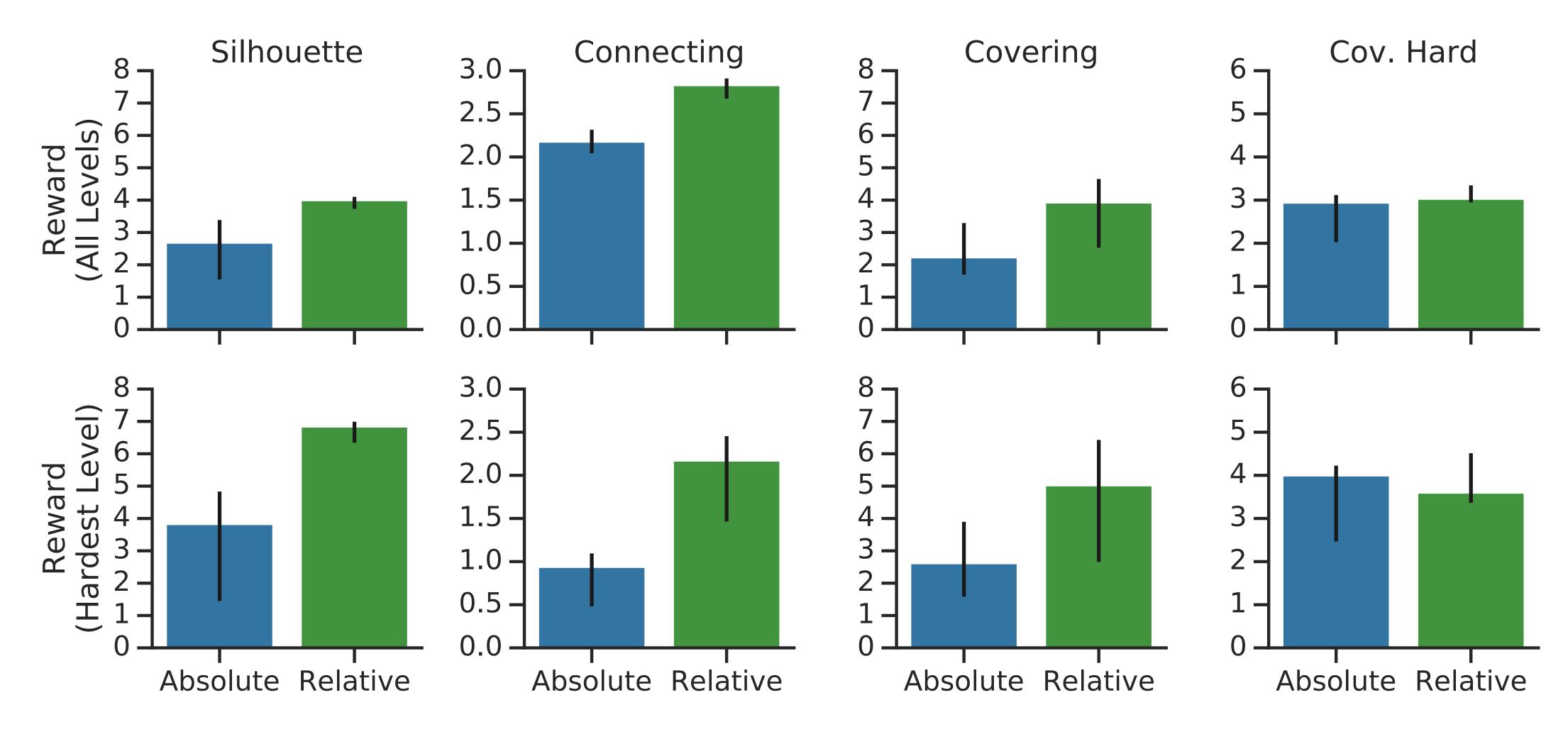


## Graph Network Agent





### Overall Results



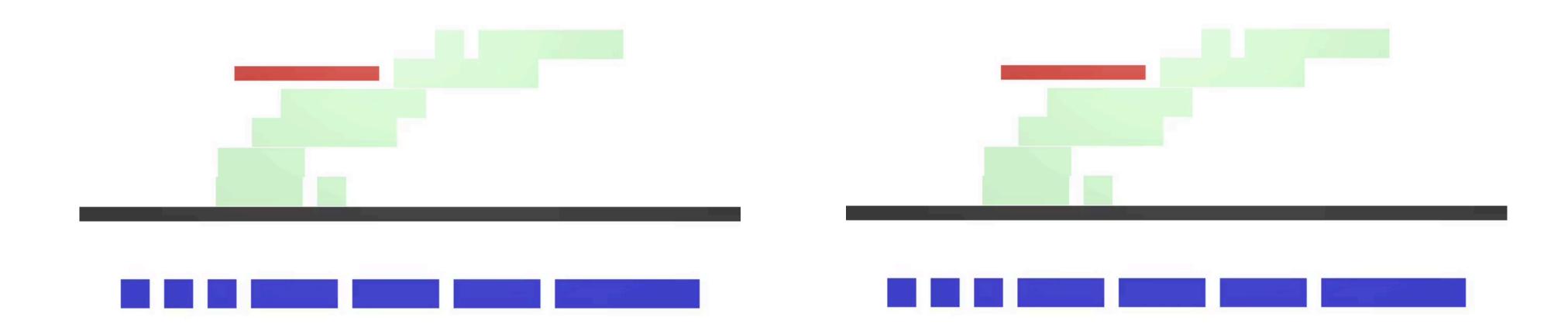


### Silhouette

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Absolute Actions (Average reward: 4.83)

Object-Centric Actions (Average reward: 6.99)



Reward: +1 per target, -0.5 per sticky block

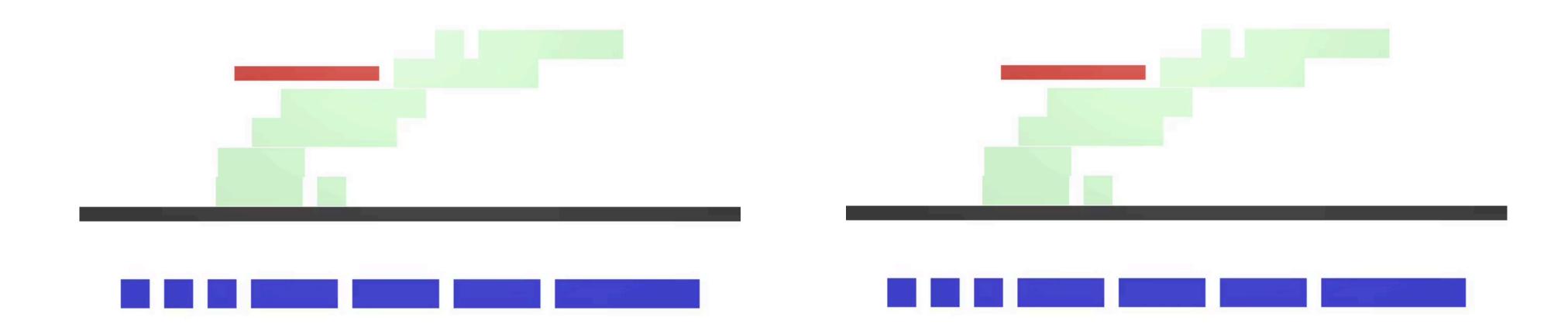


### Silhouette

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Absolute Actions (Average reward: 4.83)

Object-Centric Actions (Average reward: 6.99)



Reward: +1 per target, -0.5 per sticky block

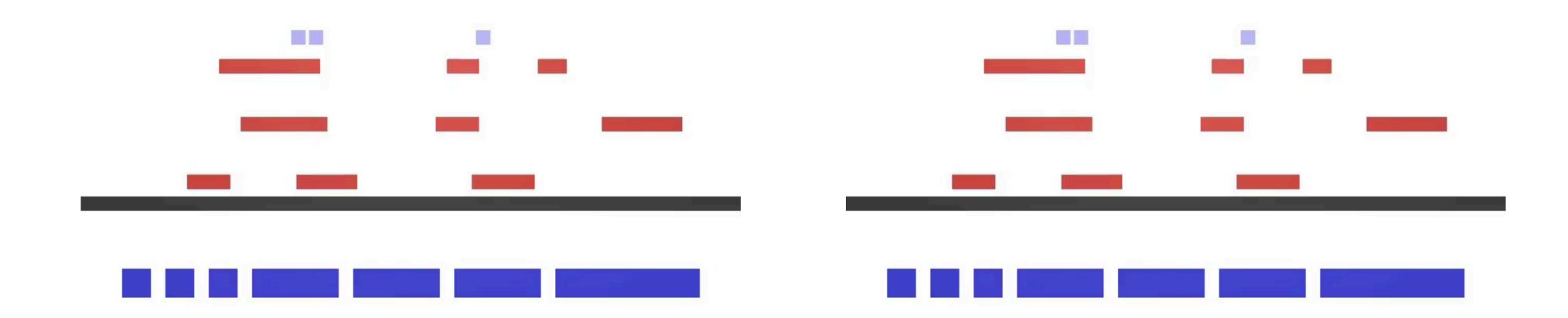


## Connecting

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Absolute Actions (Average reward: 1.09)

Object-Centric Actions (Average reward: 2.45)



Reward: +1 per target, free sticky blocks

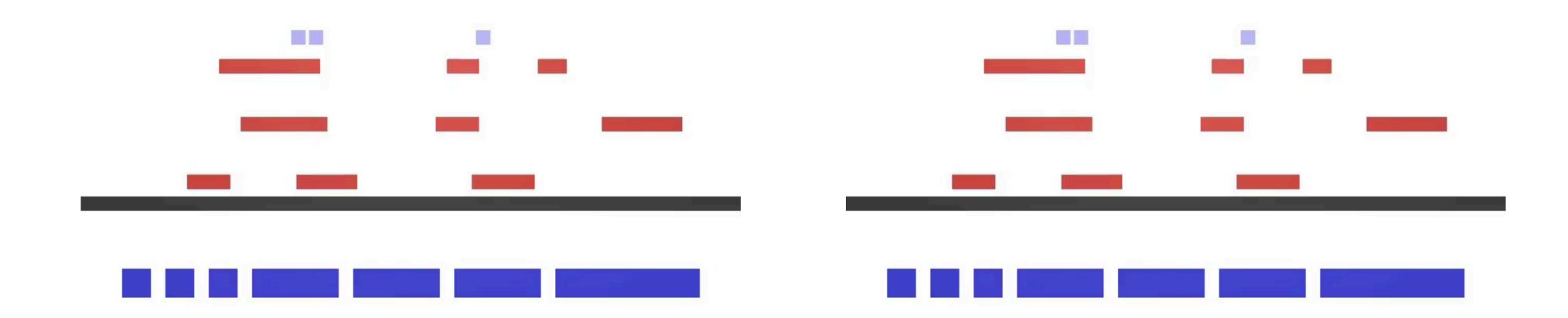


## Connecting

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Absolute Actions (Average reward: 1.09)

Object-Centric Actions (Average reward: 2.45)



Reward: +1 per target, free sticky blocks

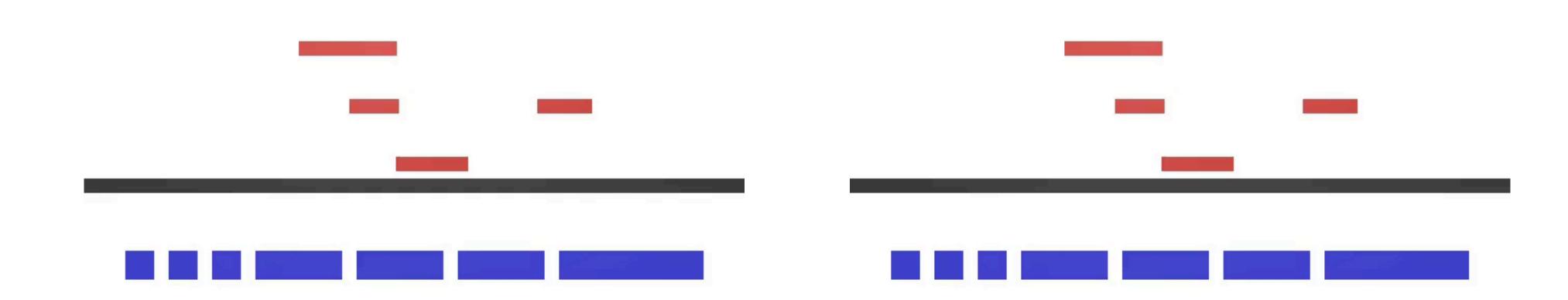


# Covering

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Absolute Actions (Average reward: 3.90)

Object-Centric Actions (Average reward: 6.43)



Reward: proportional to length covered, -2 per sticky block

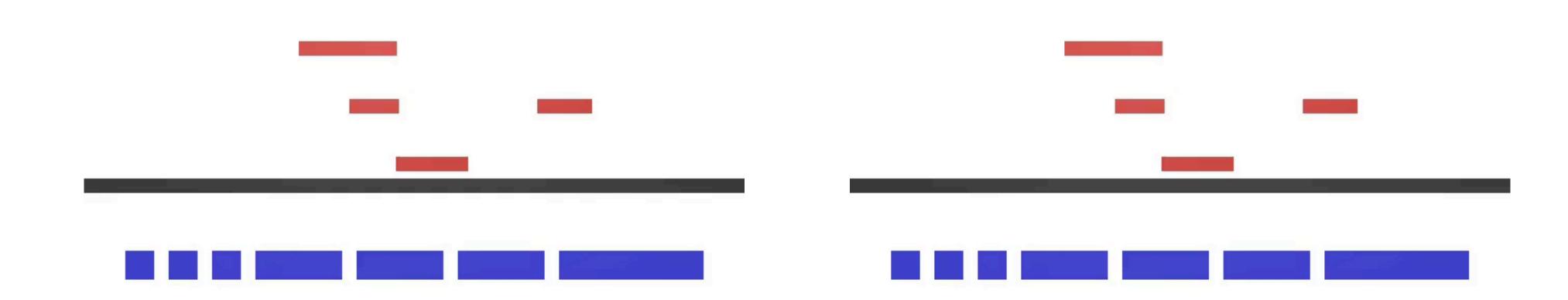


# Covering

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Absolute Actions (Average reward: 3.90)

Object-Centric Actions (Average reward: 6.43)



Reward: proportional to length covered, -2 per sticky block

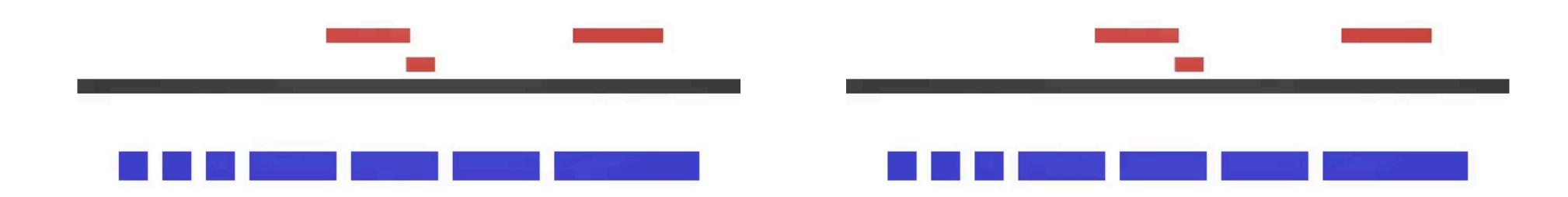


# Covering Hard

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Absolute Actions (Average reward: 4.22)

Object-Centric Actions (Average reward: 4.51)



Reward: proportional to length covered, -0.5 per sticky block

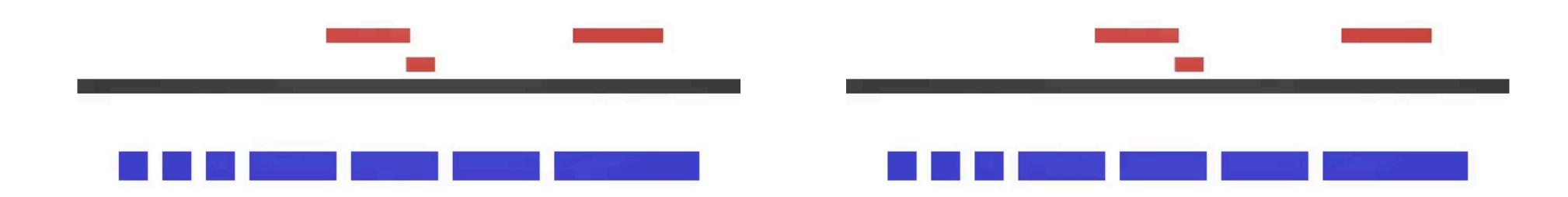


# Covering Hard

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Absolute Actions (Average reward: 4.22)

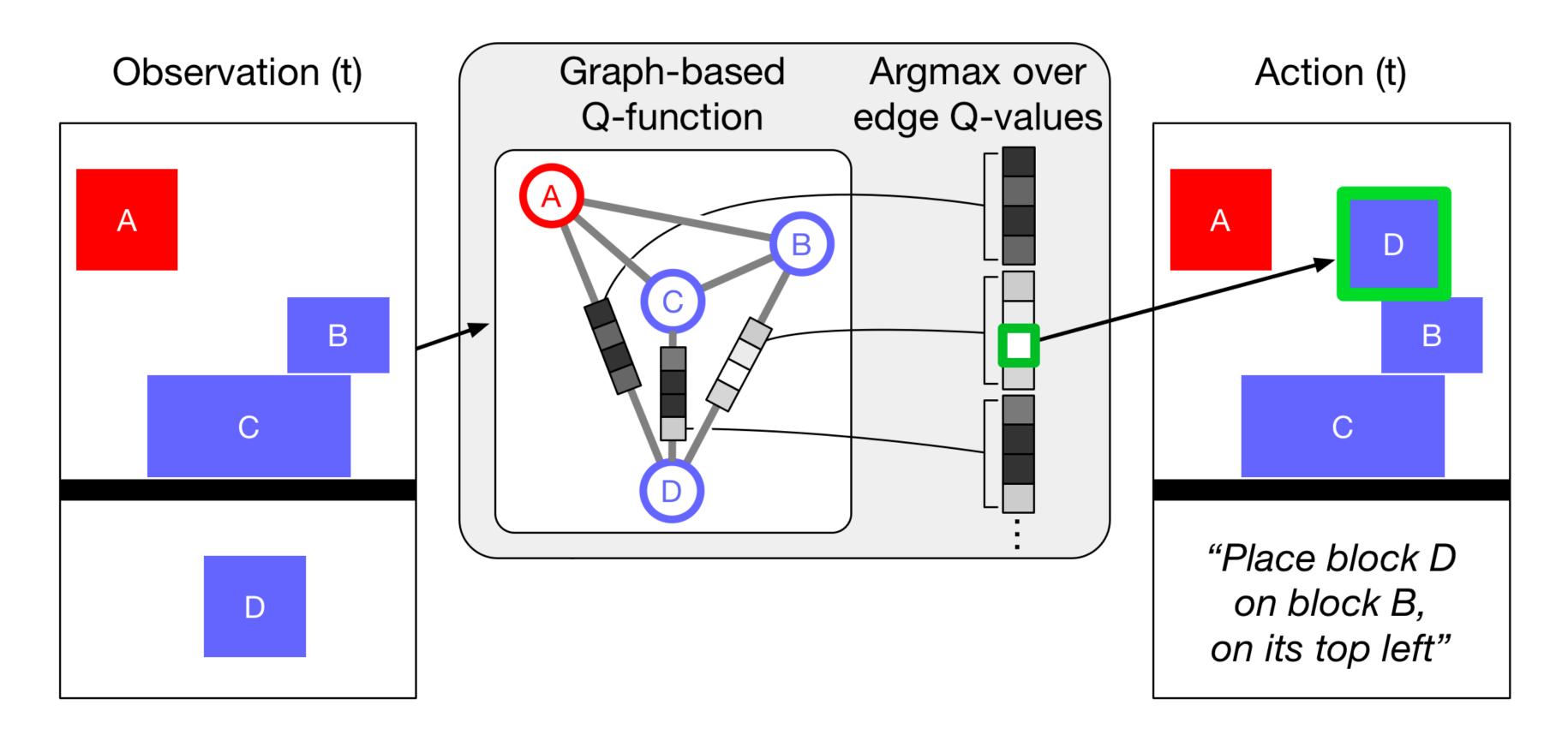
Object-Centric Actions (Average reward: 4.51)



Reward: proportional to length covered, -0.5 per sticky block

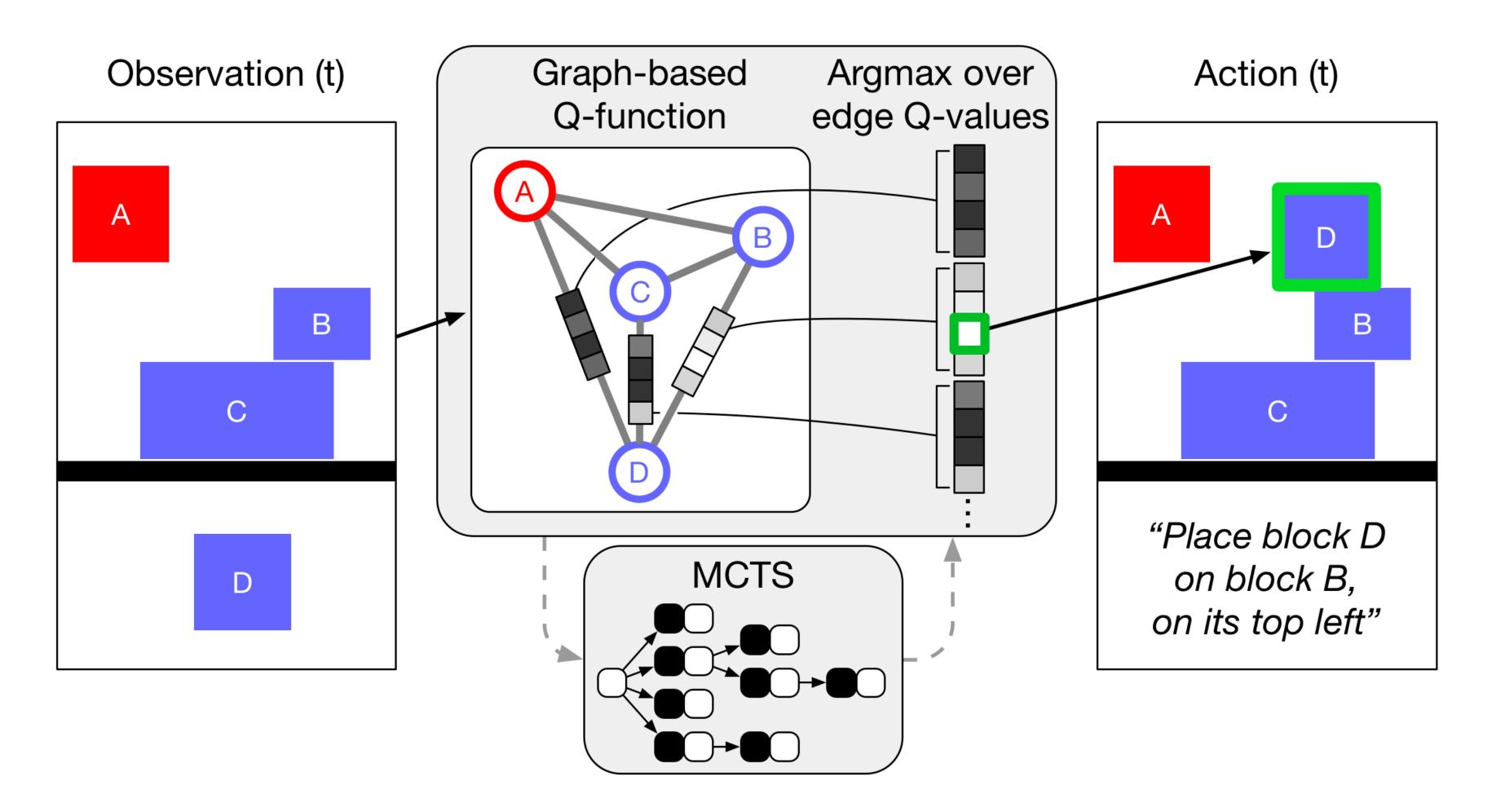


## Graph Network Agent



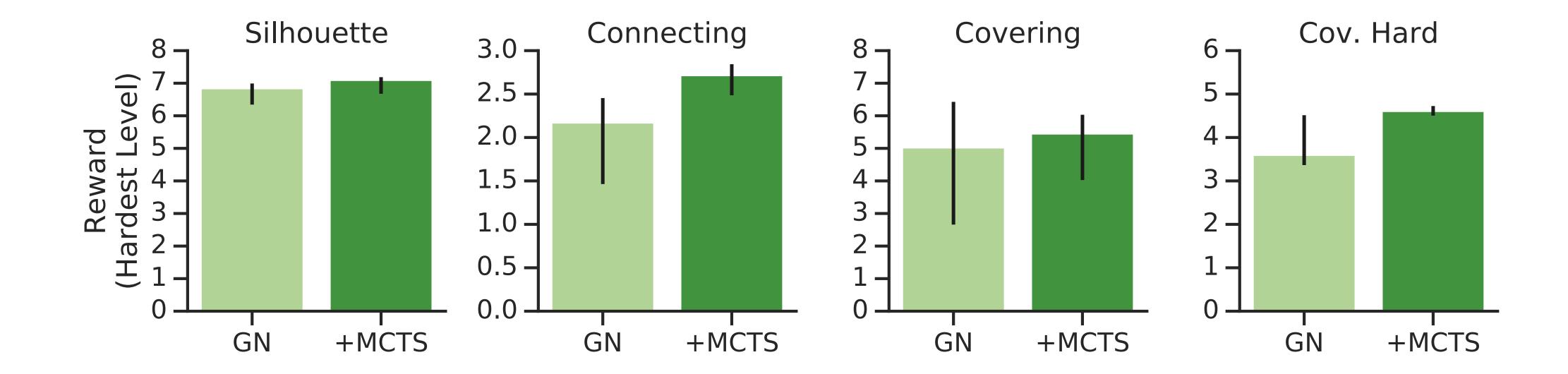


## Graph Network Agent





## Results: Planning

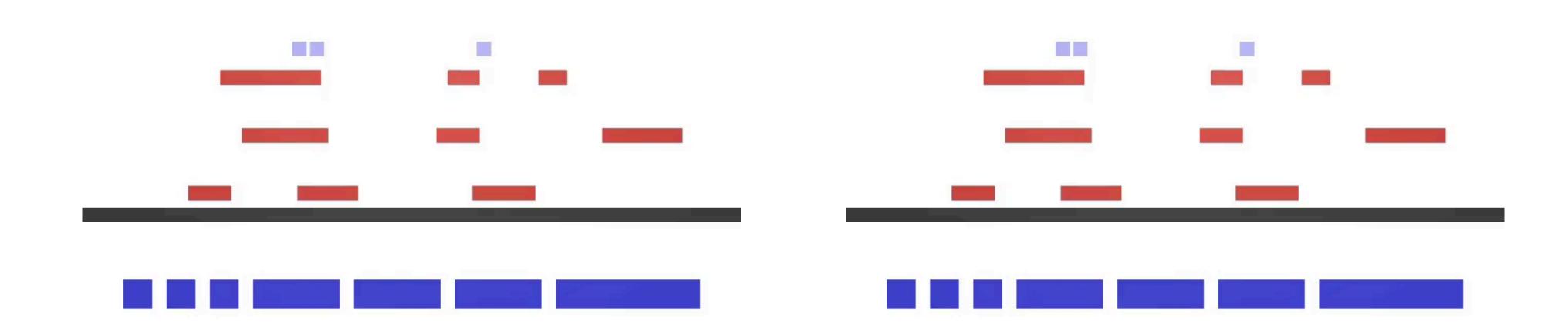


## Results: Planning

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Object-Centric Actions (Average reward: 2.45)

Object-Centric Actions +MCTS (Average reward: 2.84)



Reward: +1 per target, free sticky blocks

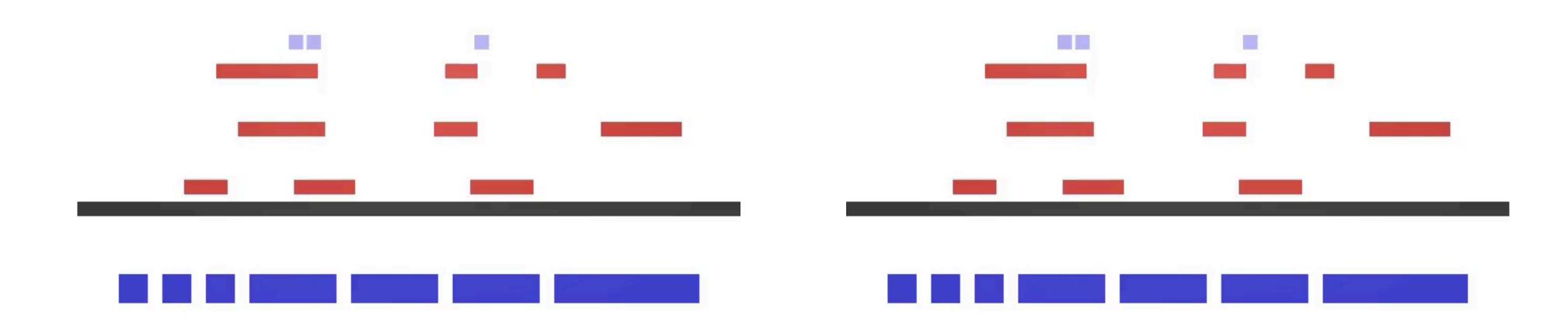


# Results: Planning

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

Object-Centric Actions (Average reward: 2.45)

Object-Centric Actions +MCTS (Average reward: 2.84)

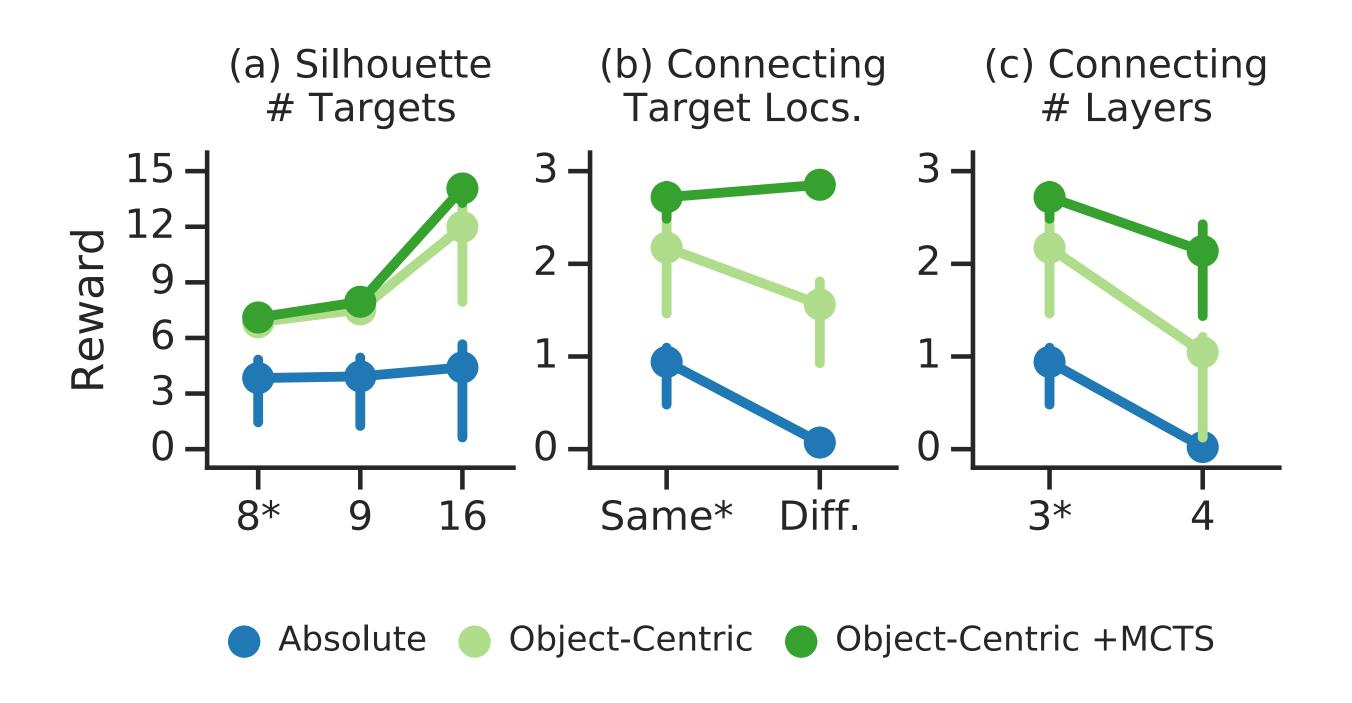


Reward: +1 per target, free sticky blocks



#### Results: Generalization

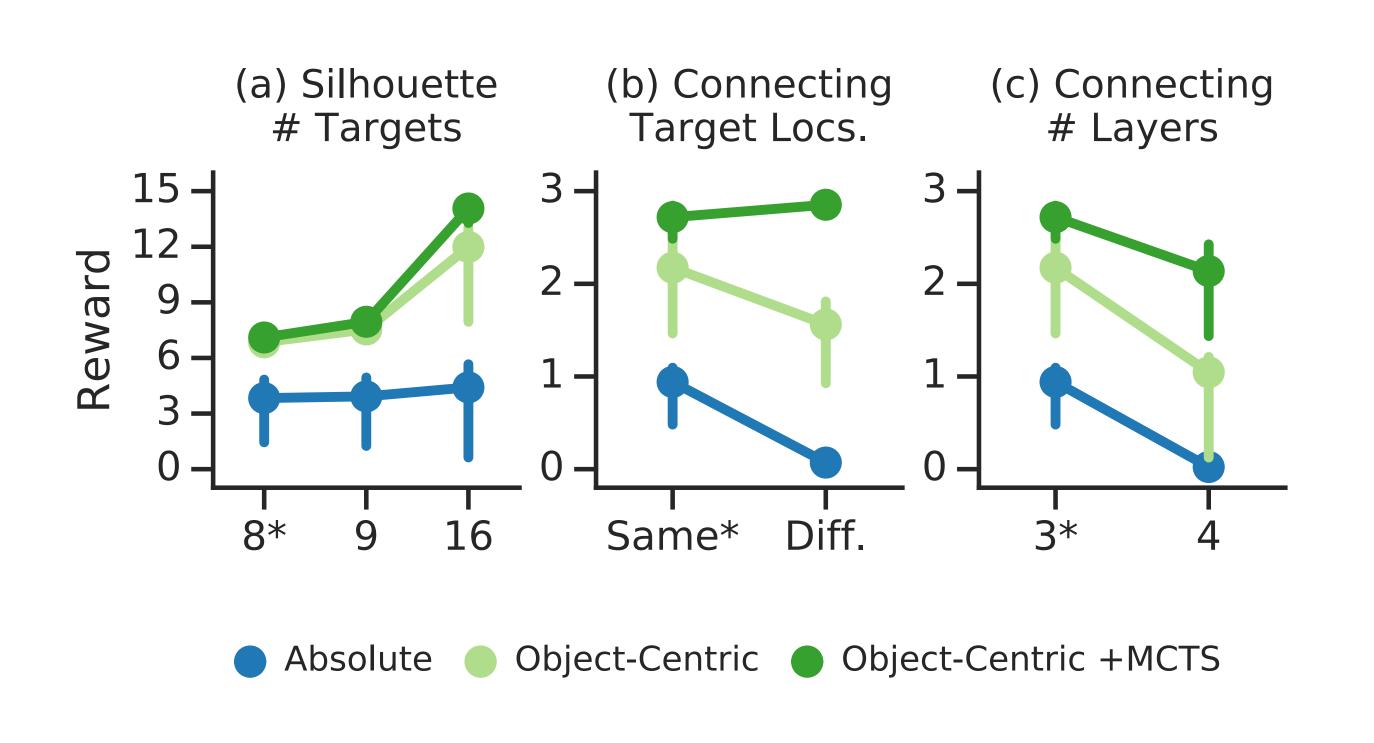
Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

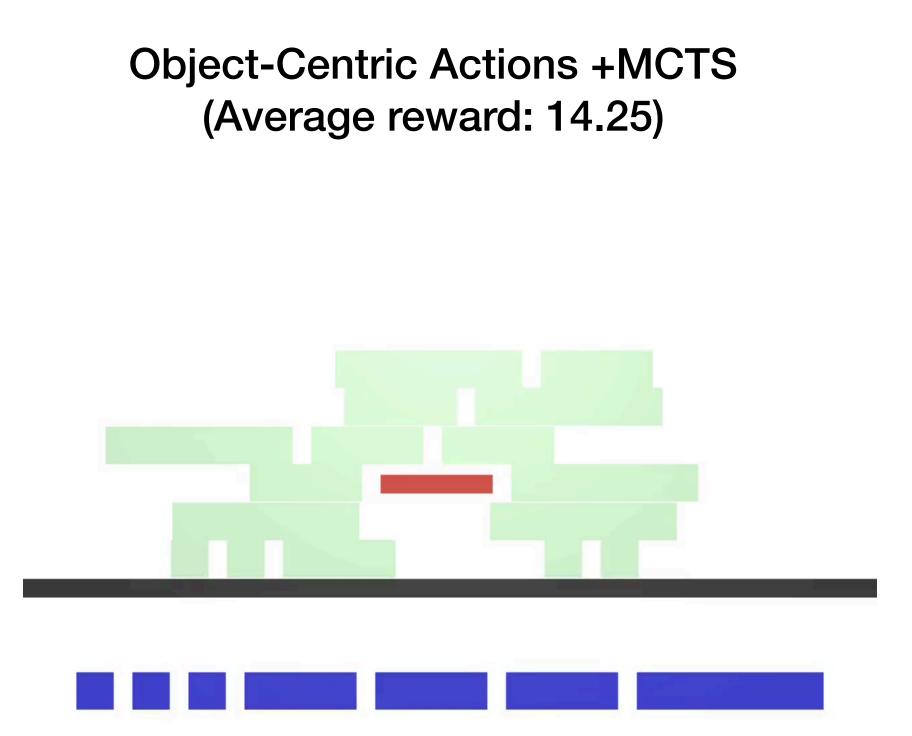




#### Results: Generalization

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)

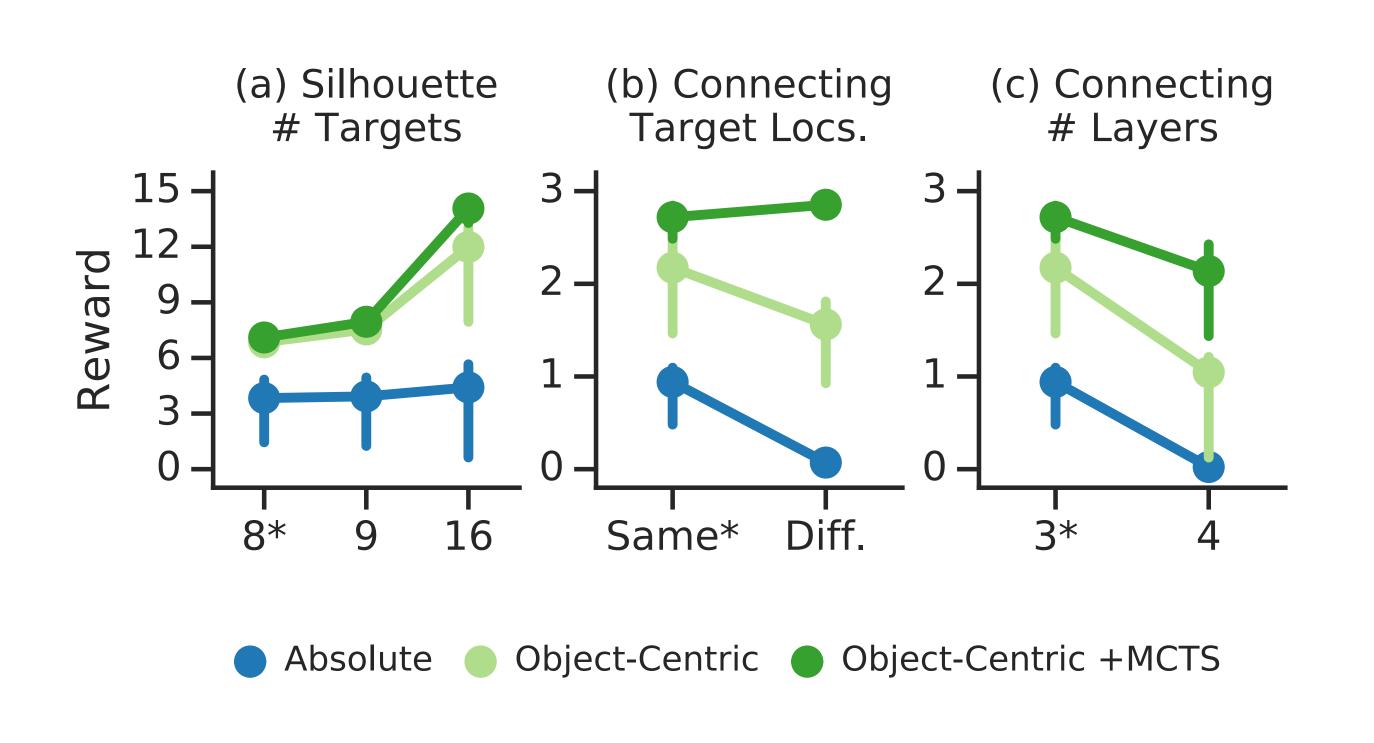


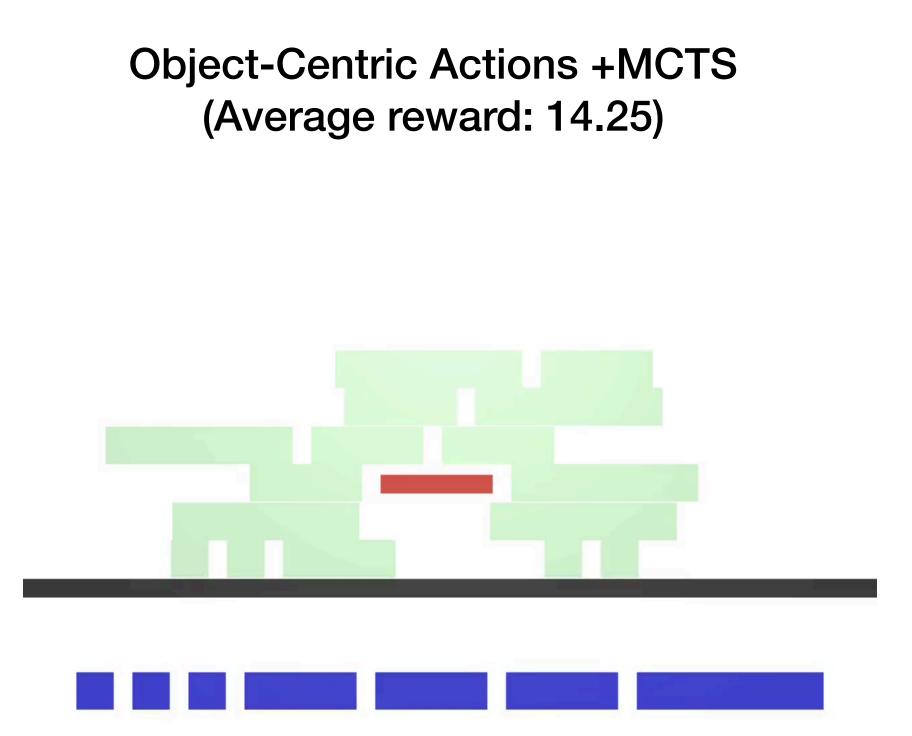




#### Results: Generalization

Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia & Hamrick (ICML, 2019)



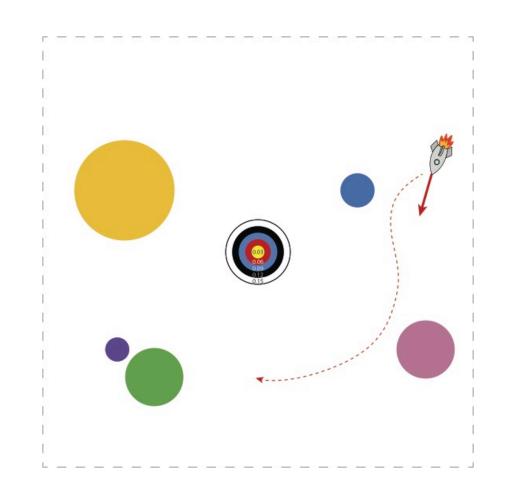




#### Outline

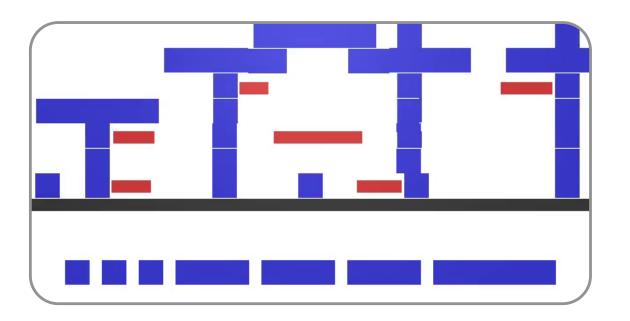
1. **Structured Computation**: the way that individual computations or functions are composed into more complex structures.

Hamrick, Ballard, Pascanu, Vinyals, Heess, & Battaglia (2017, ICLR)



2. Structured Representation: the format of the data that computations are performed over.

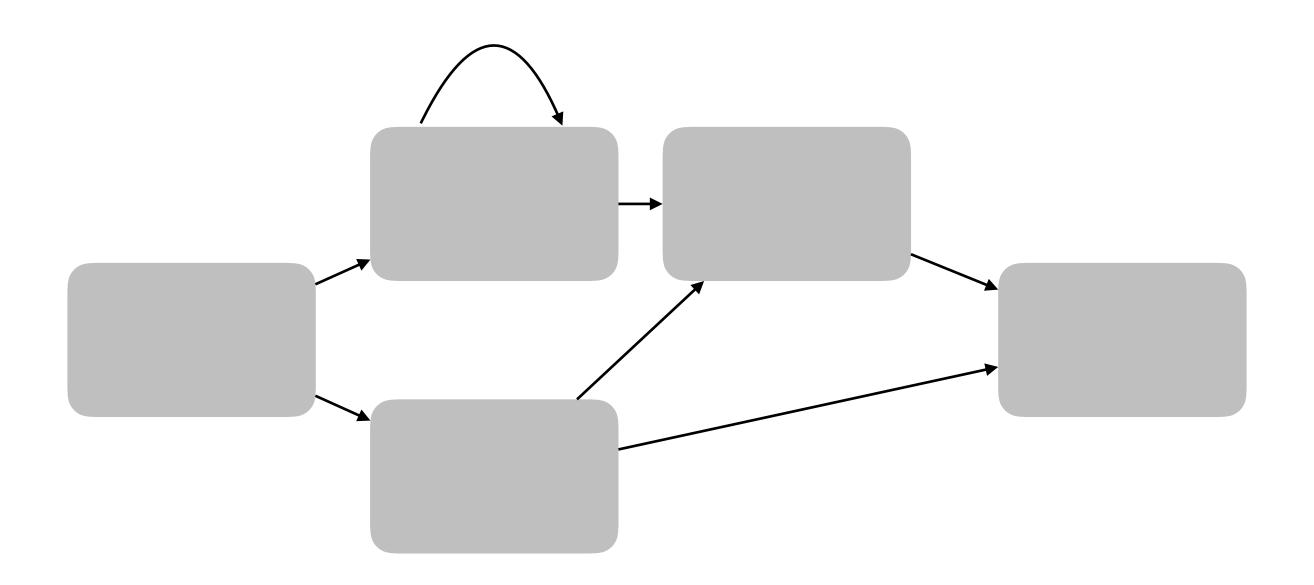
Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (2018, arXiv) Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kohli, Battaglia, & Hamrick (2019, ICML)



Flexibility, adaptivity and generalization are about having the right structure.

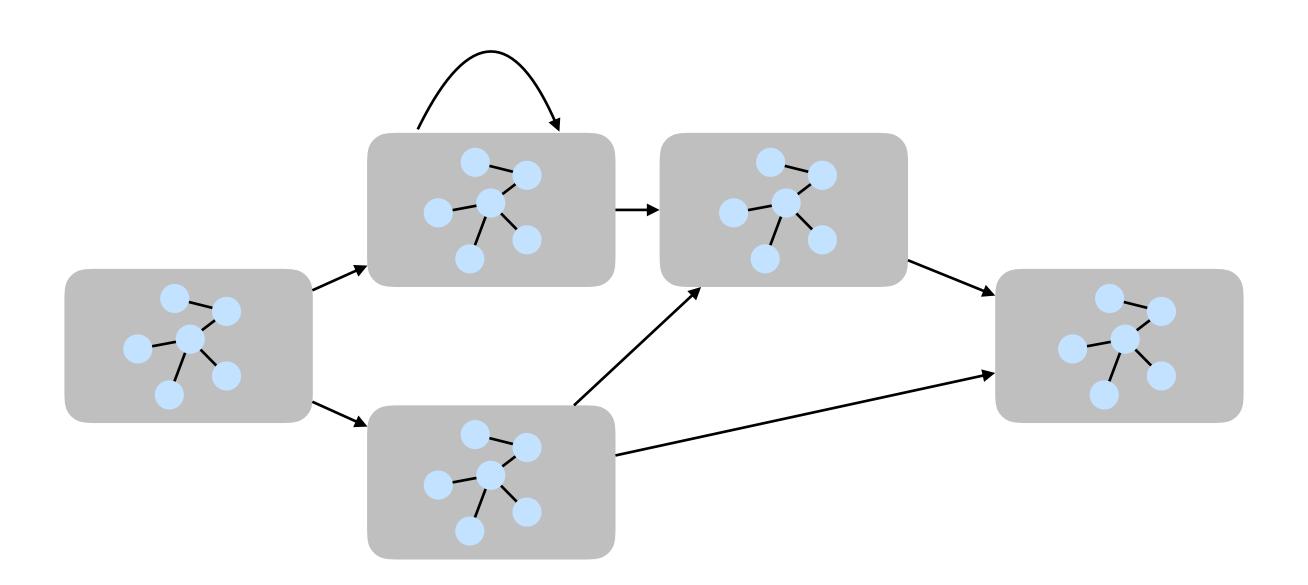


Flexibility, adaptivity and generalization are about having the right structure.





# Flexibility, adaptivity and generalization are about having the right structure.



#### Thanks!

Andy Ballard

Victor Bapst

Peter Battaglia

Hanjun Dai

Carl Doersch

Nicolas Heess

Thomas Kipf

Pushmeet Kohli

Yujia Li

Razvan Pascanu

Tobias Pfaff

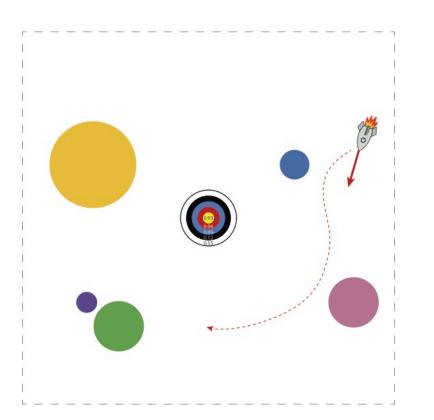
David Reichert

Alvaro Sanchez-Gonzalez

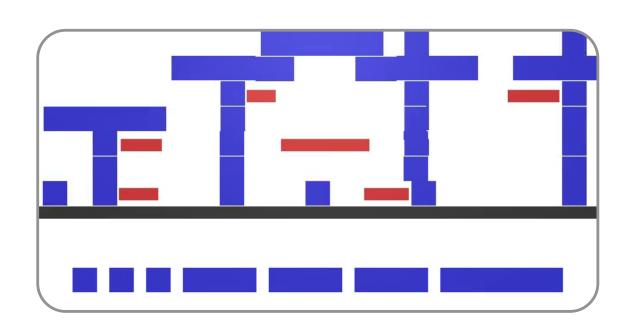
Kim Stachenfeld

Oriol Vinyals

Theo Weber

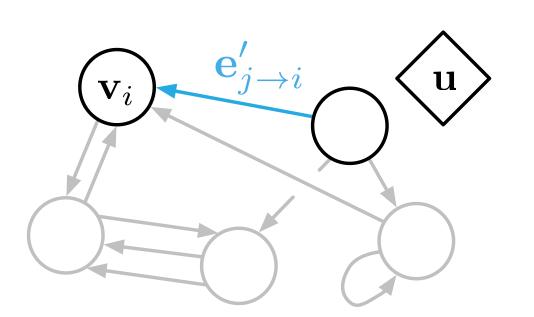


Hamrick, Ballard, Pascanu, Vinyals, Heess, & Battaglia (*ICLR* 2017)



Bapst\*, Sanchez-Gonzalez\*, Doersch, Stachenfeld, Kolhli, Battaglia, & Hamrick (*ICML* 2019)

\*equal contribution

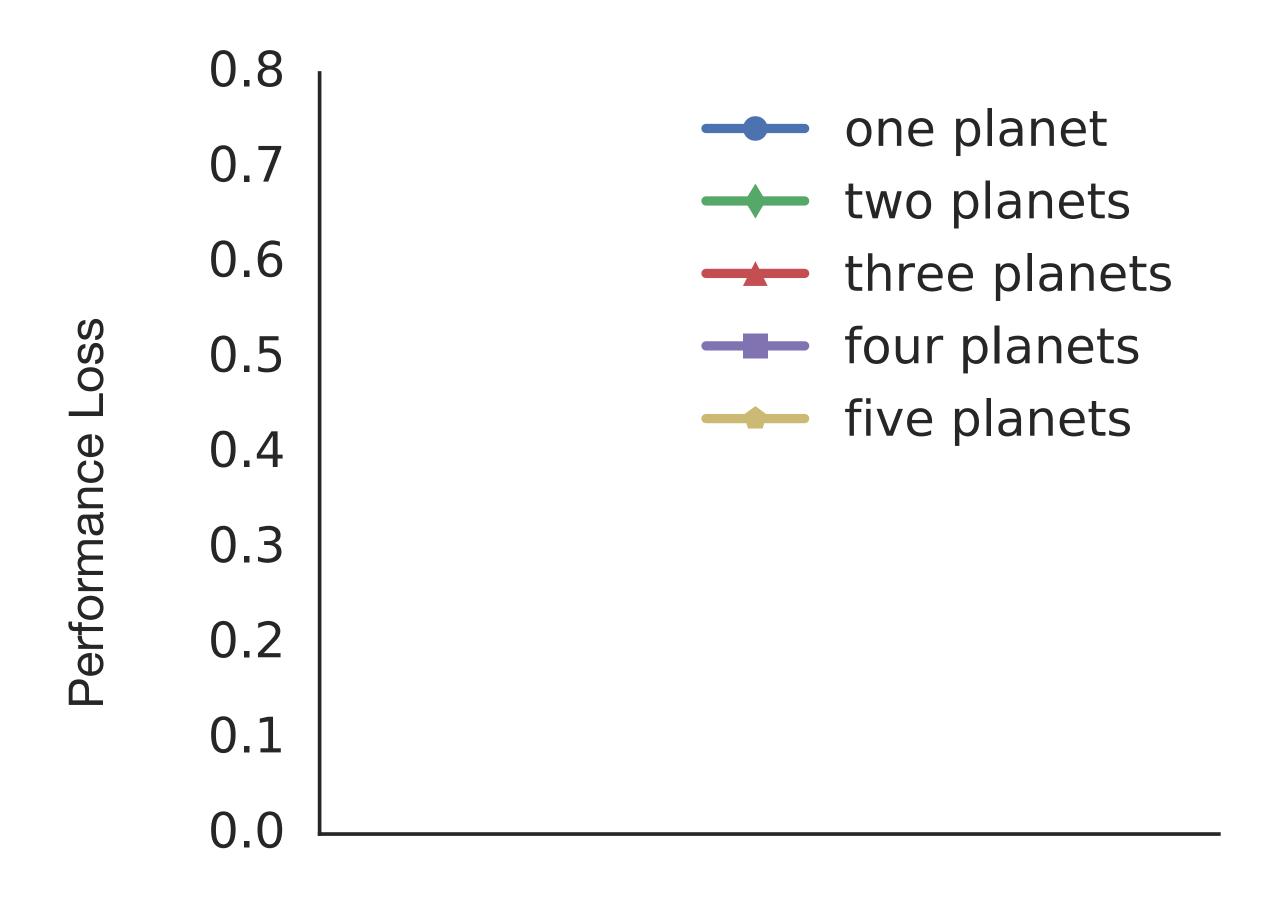


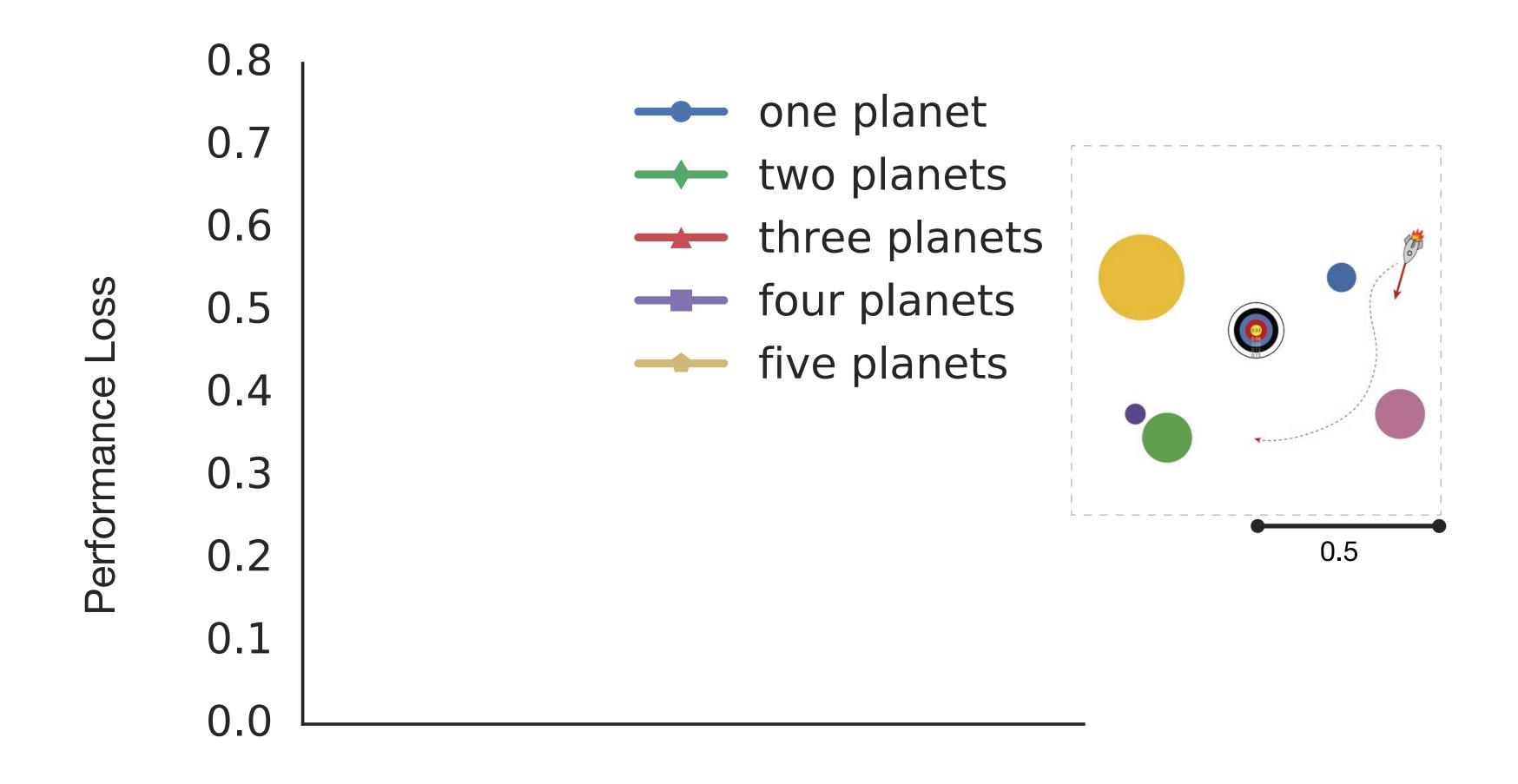
Battaglia, Hamrick, Bapst, Sanchez-Gonzalez, Zambaldi, et al. (arXiv 2018)

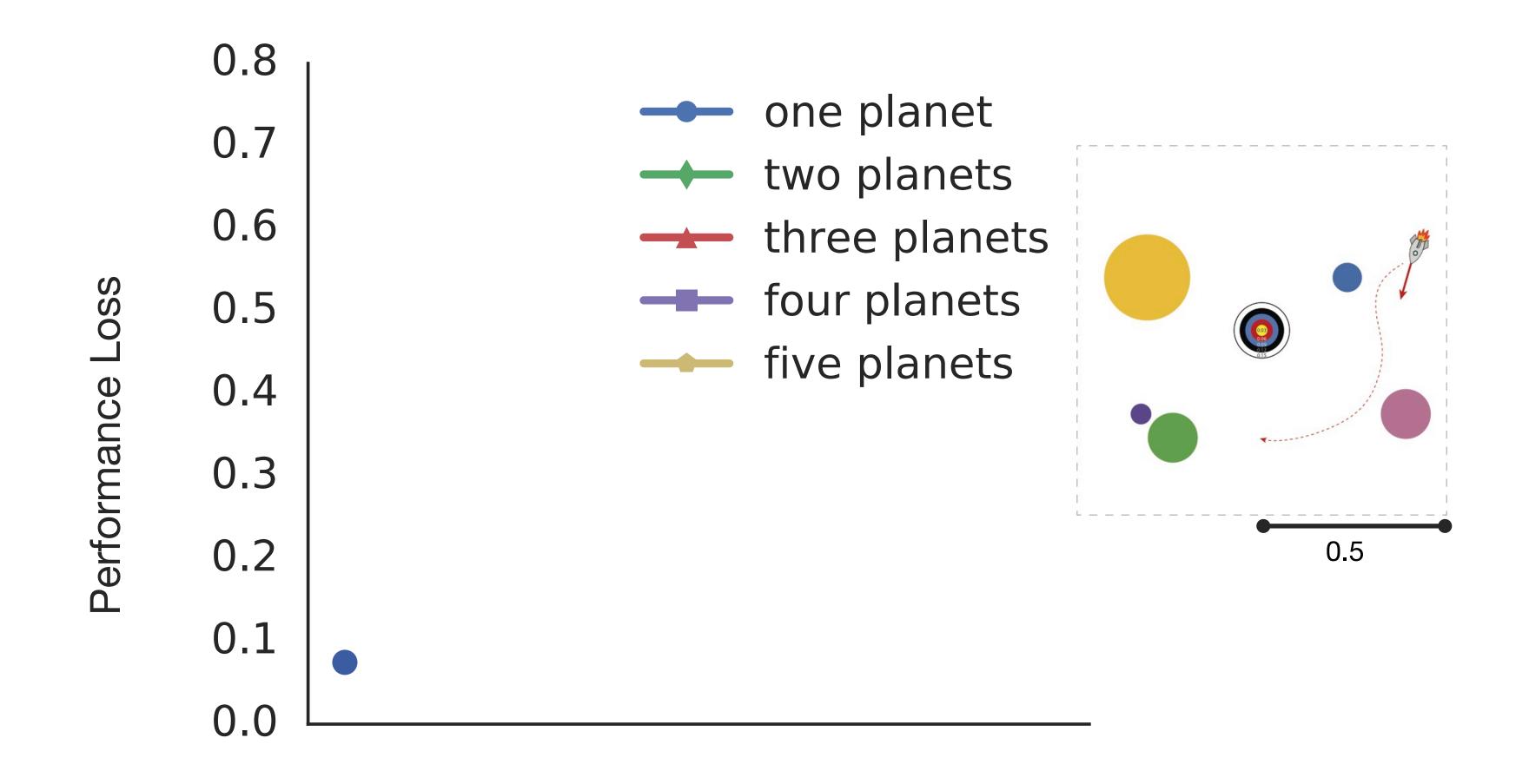


### Extra Slides

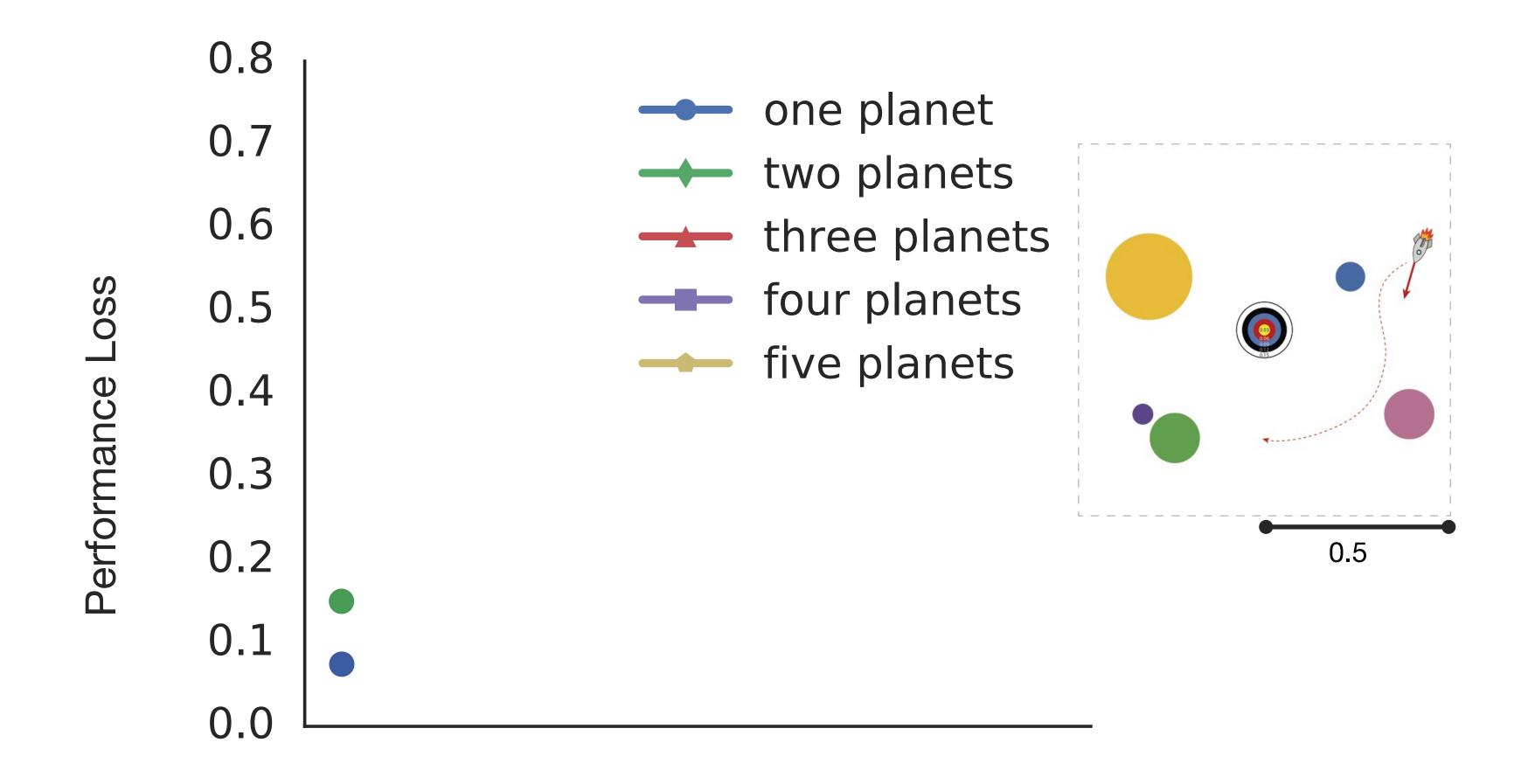




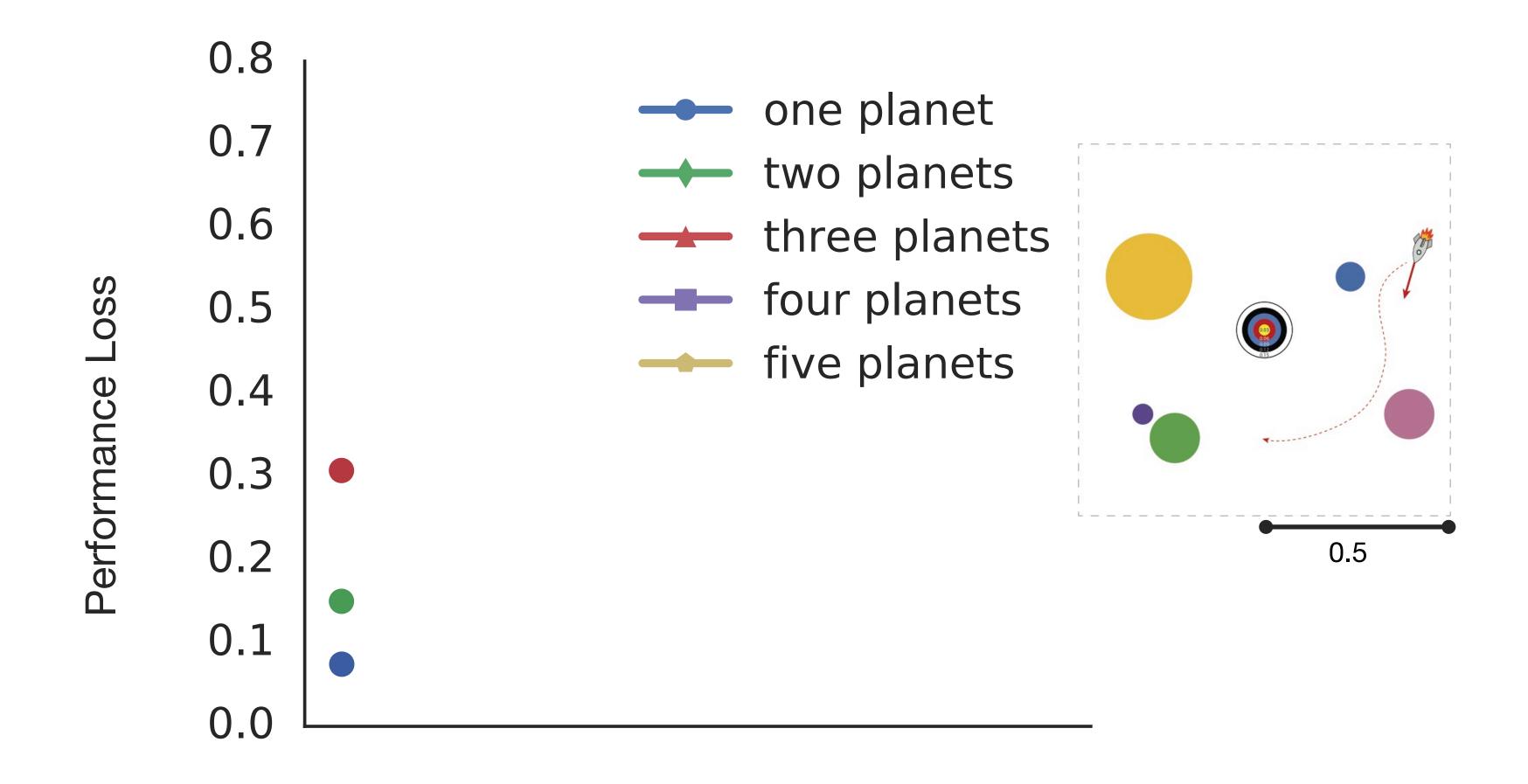




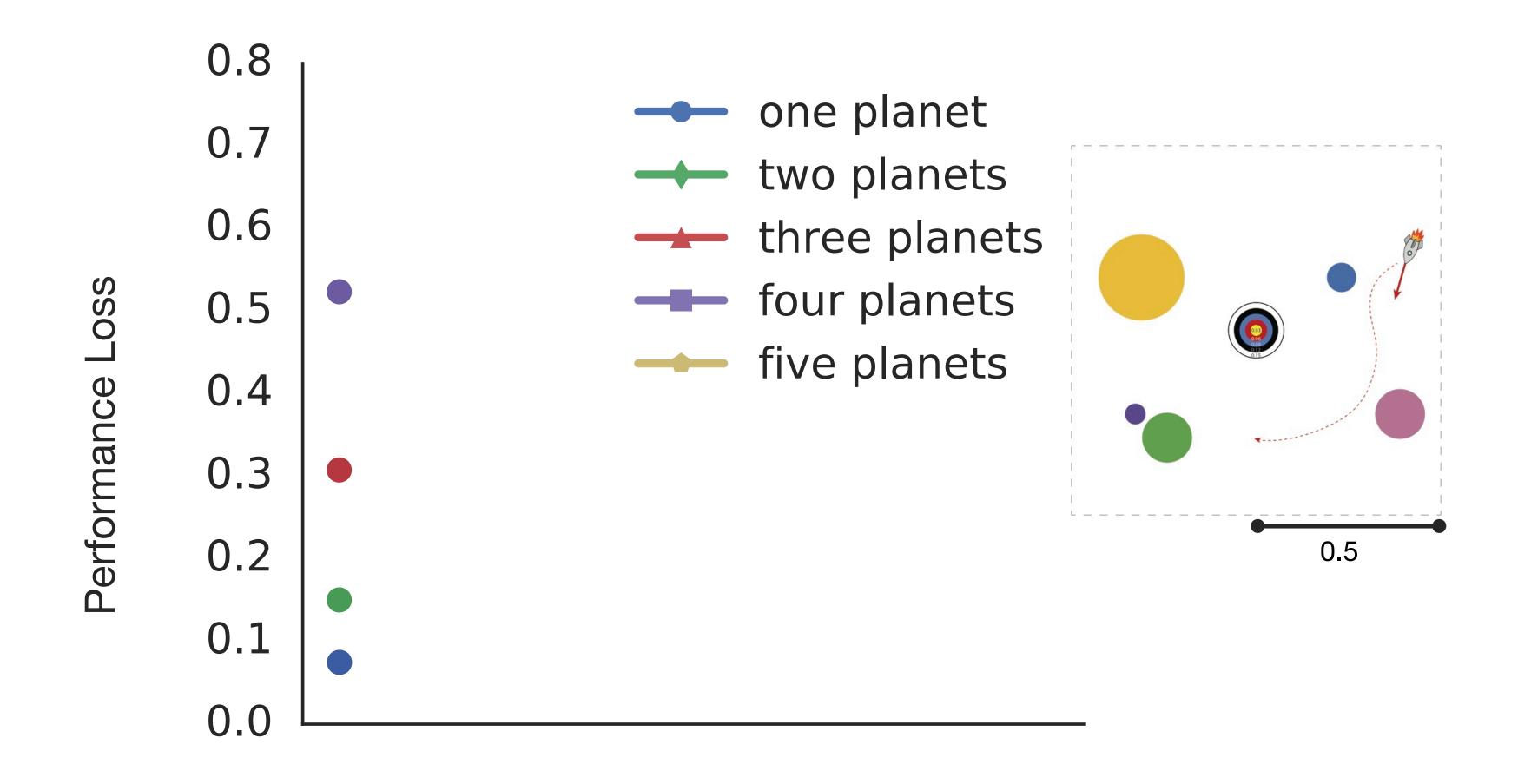


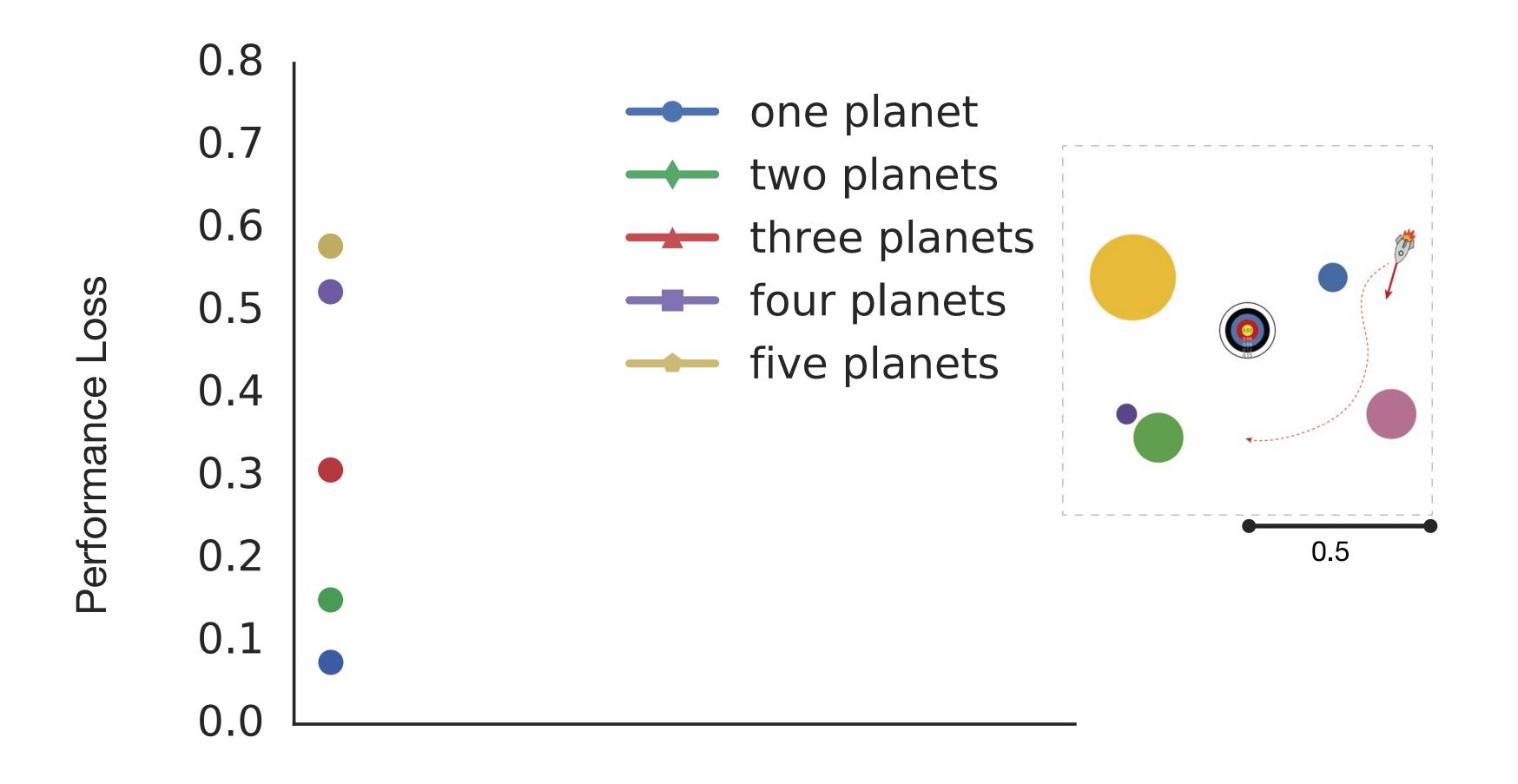


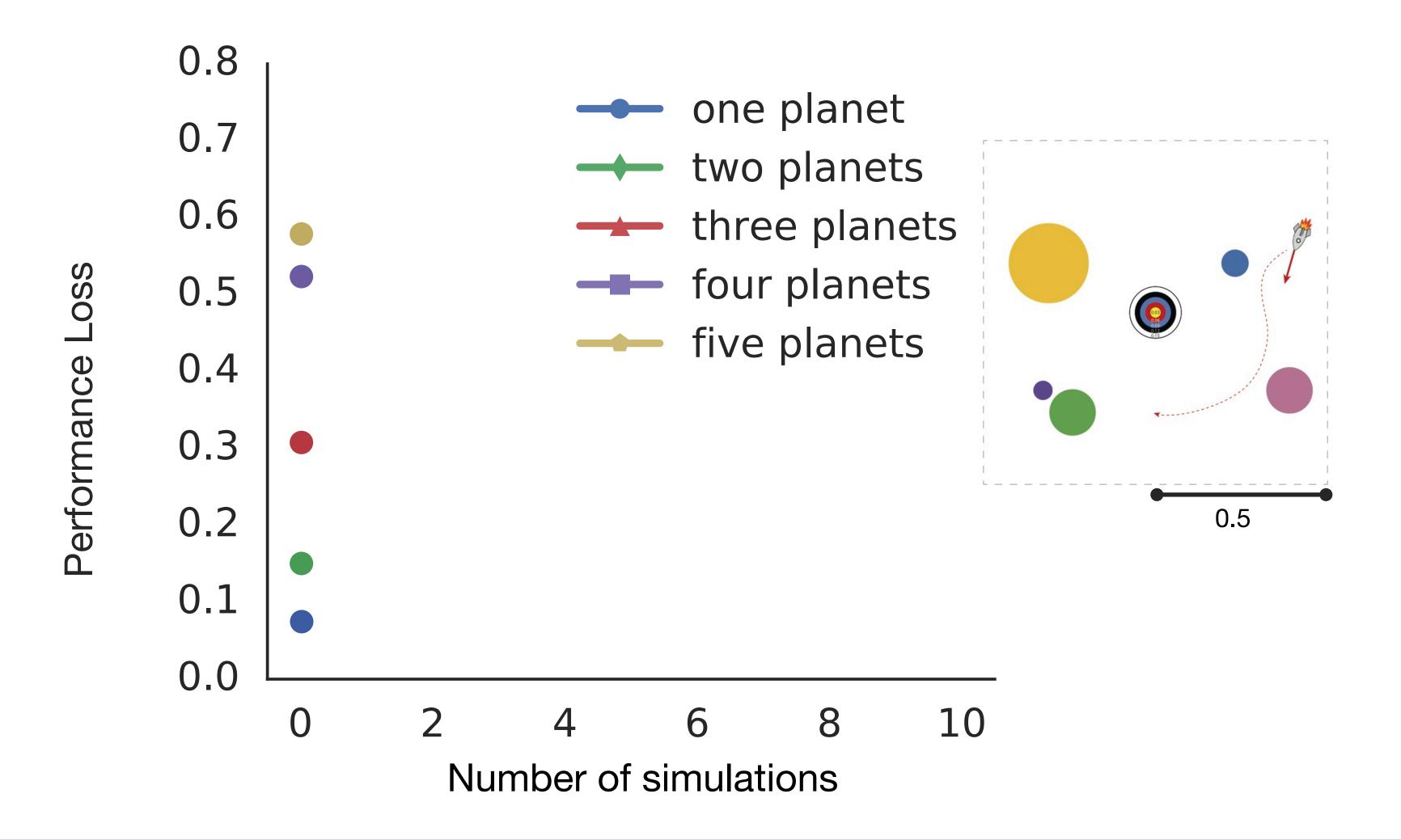




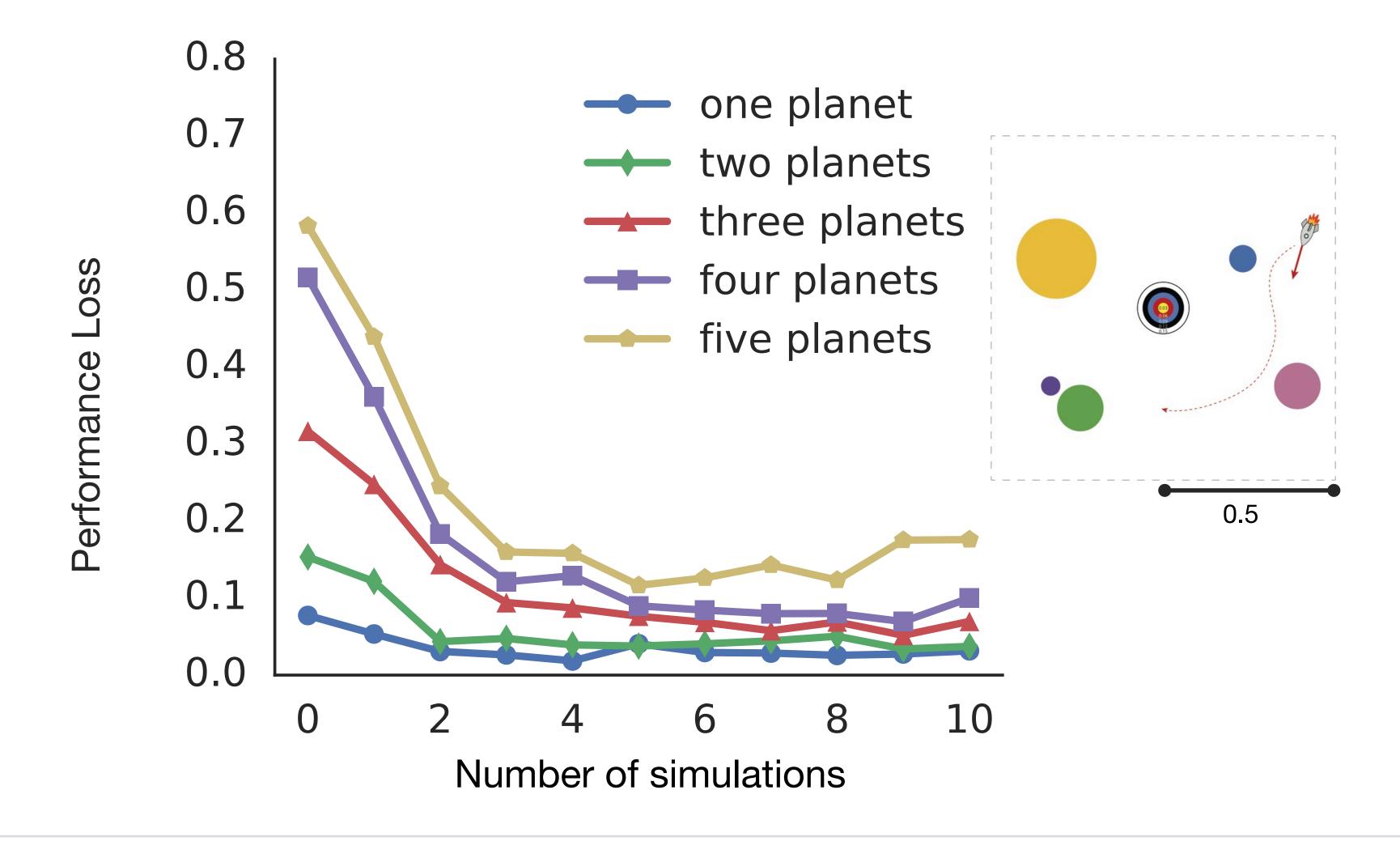






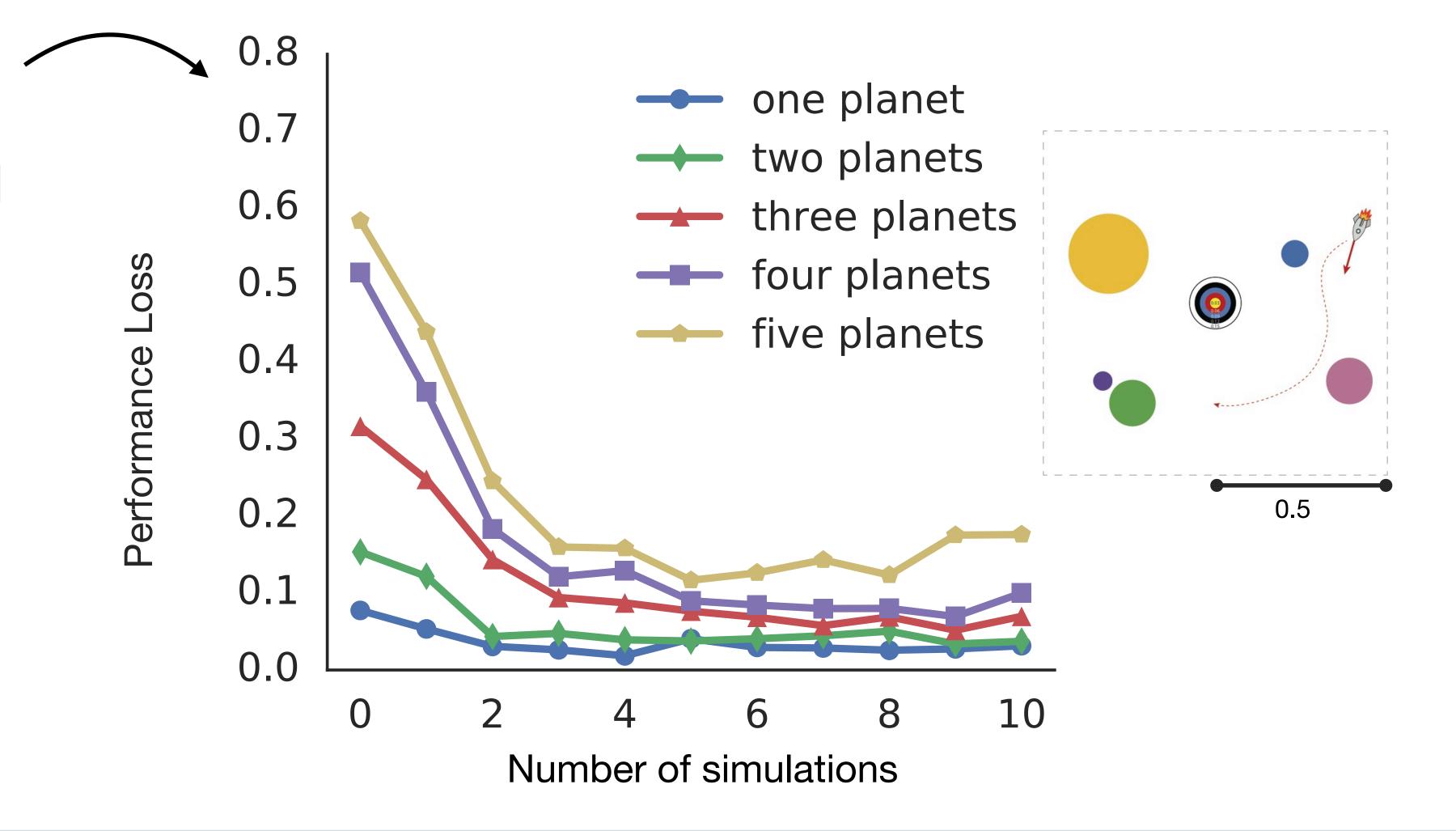




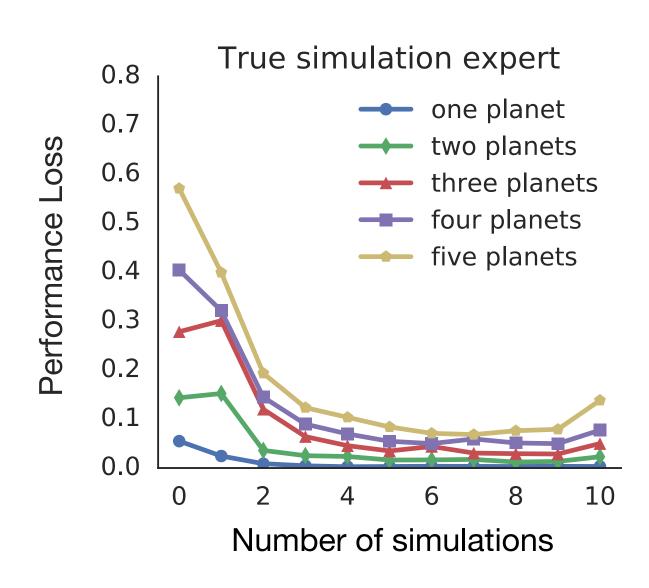




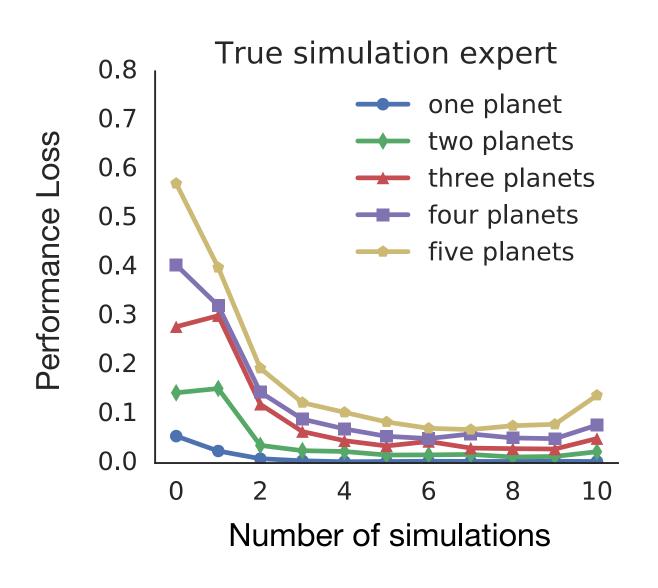
This is using a learned expert, which is trained simultaneously with the agent

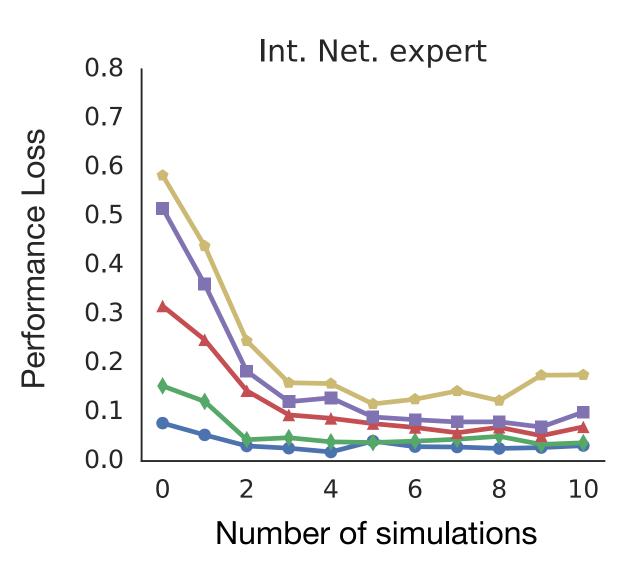




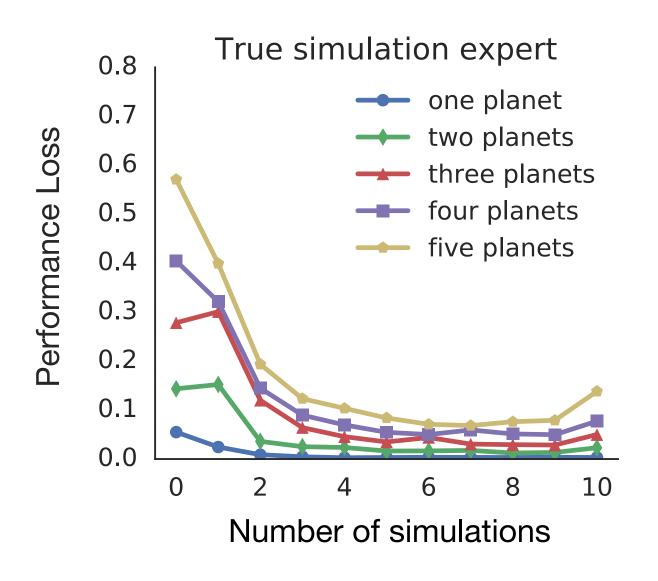


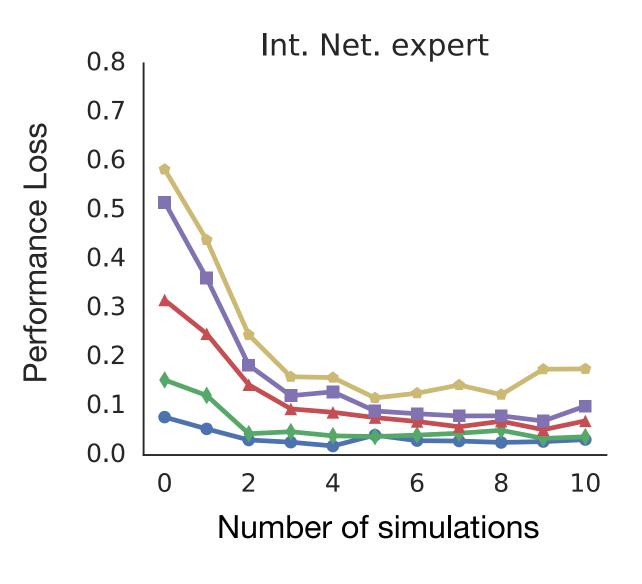


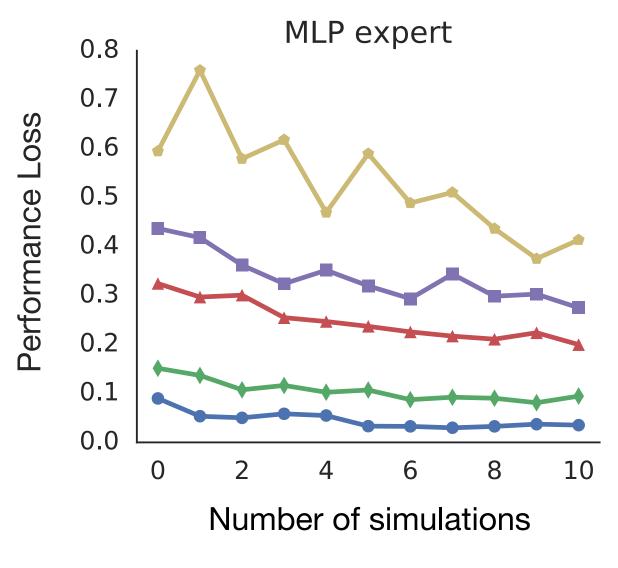








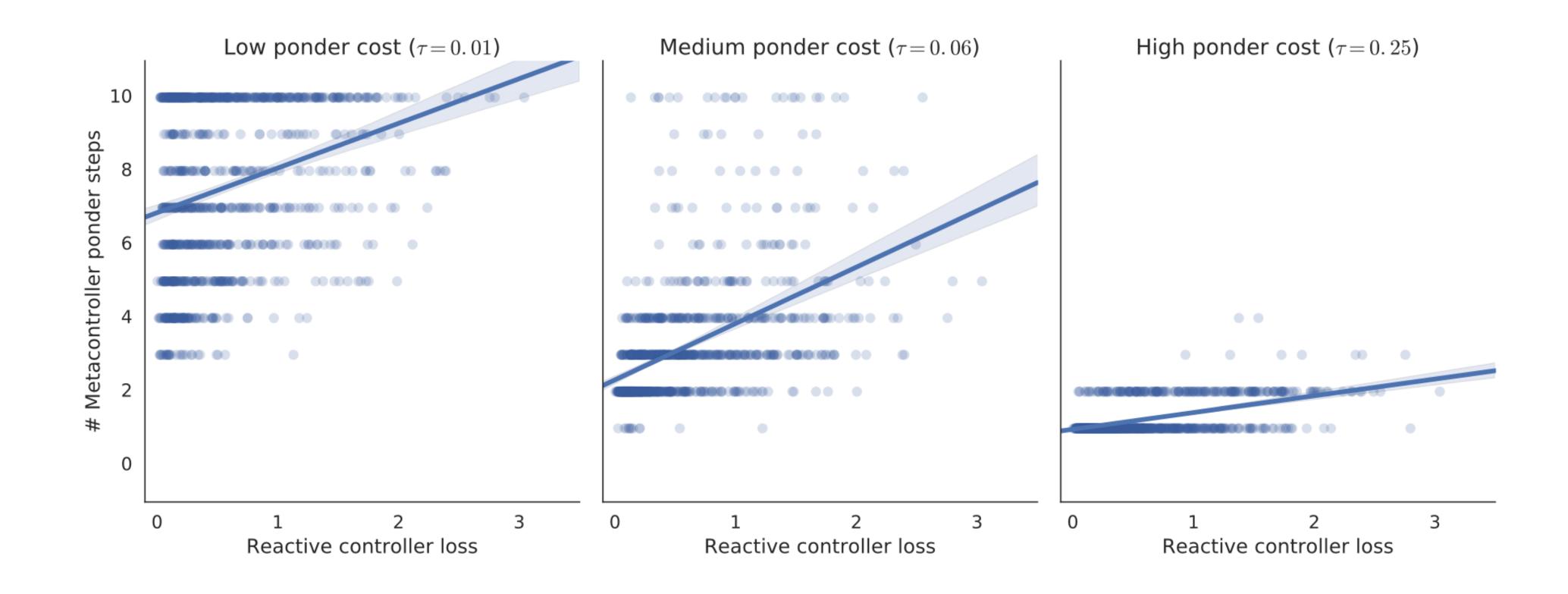




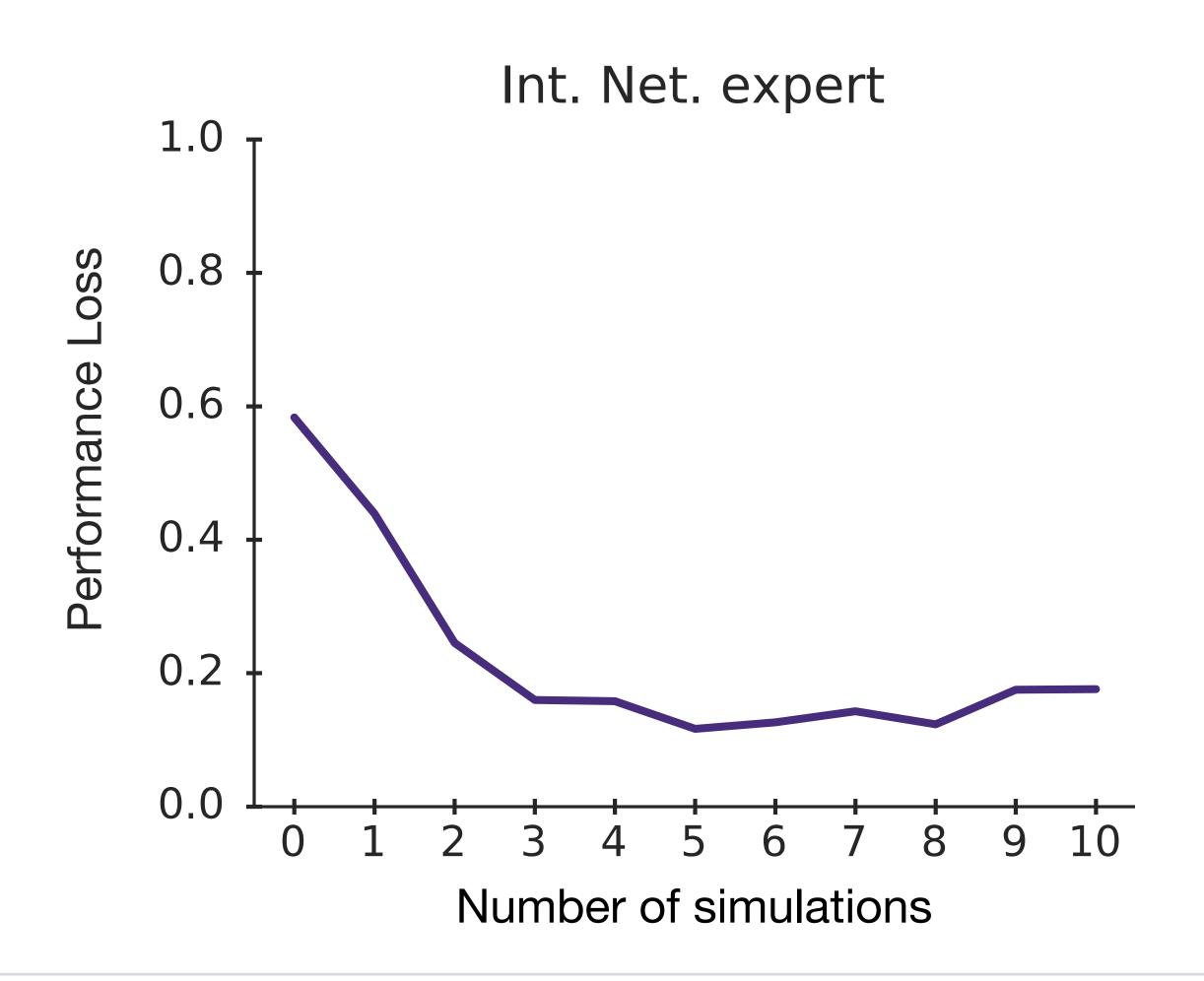
Effect of difficulty (IN expert)



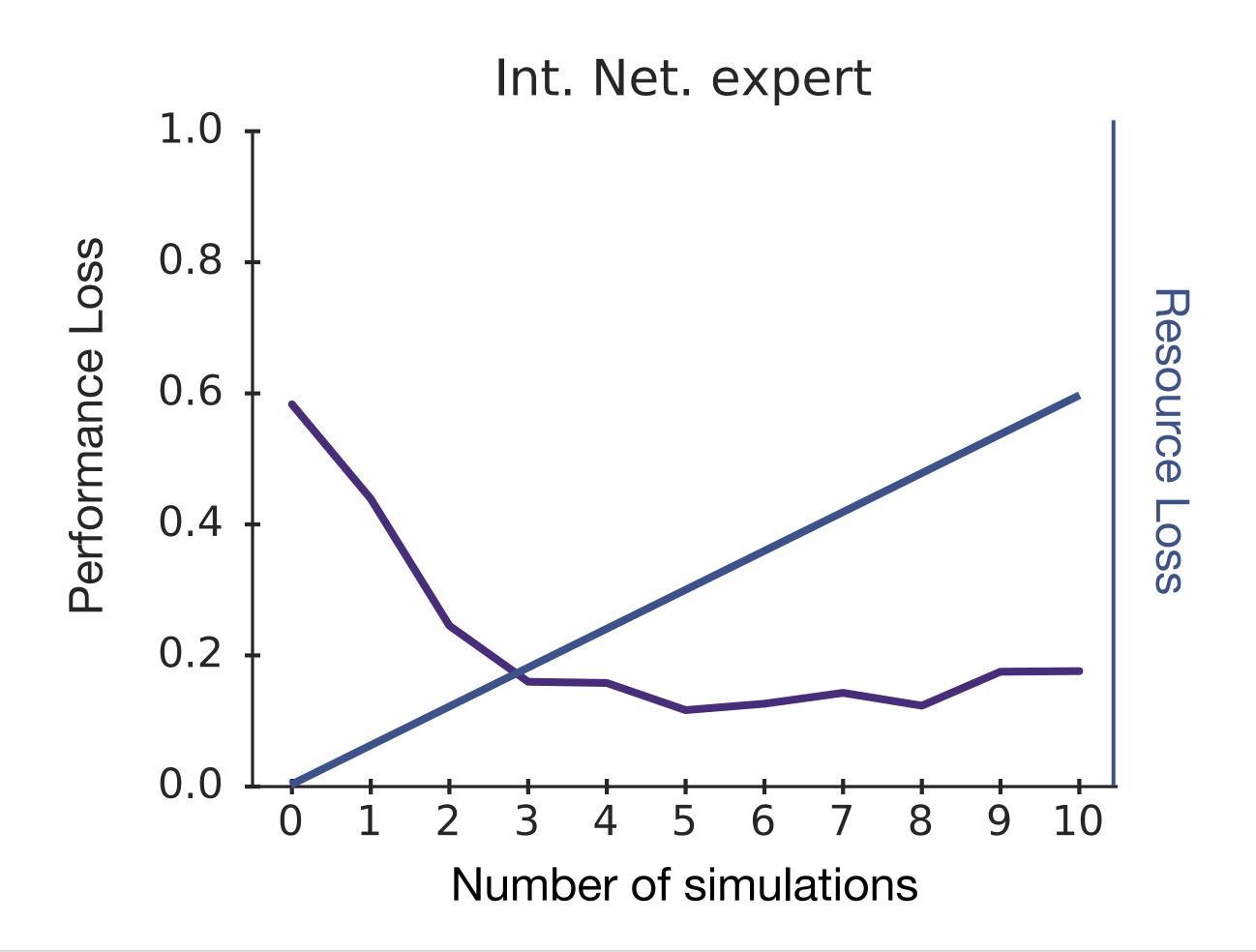
Effect of difficulty (IN expert)



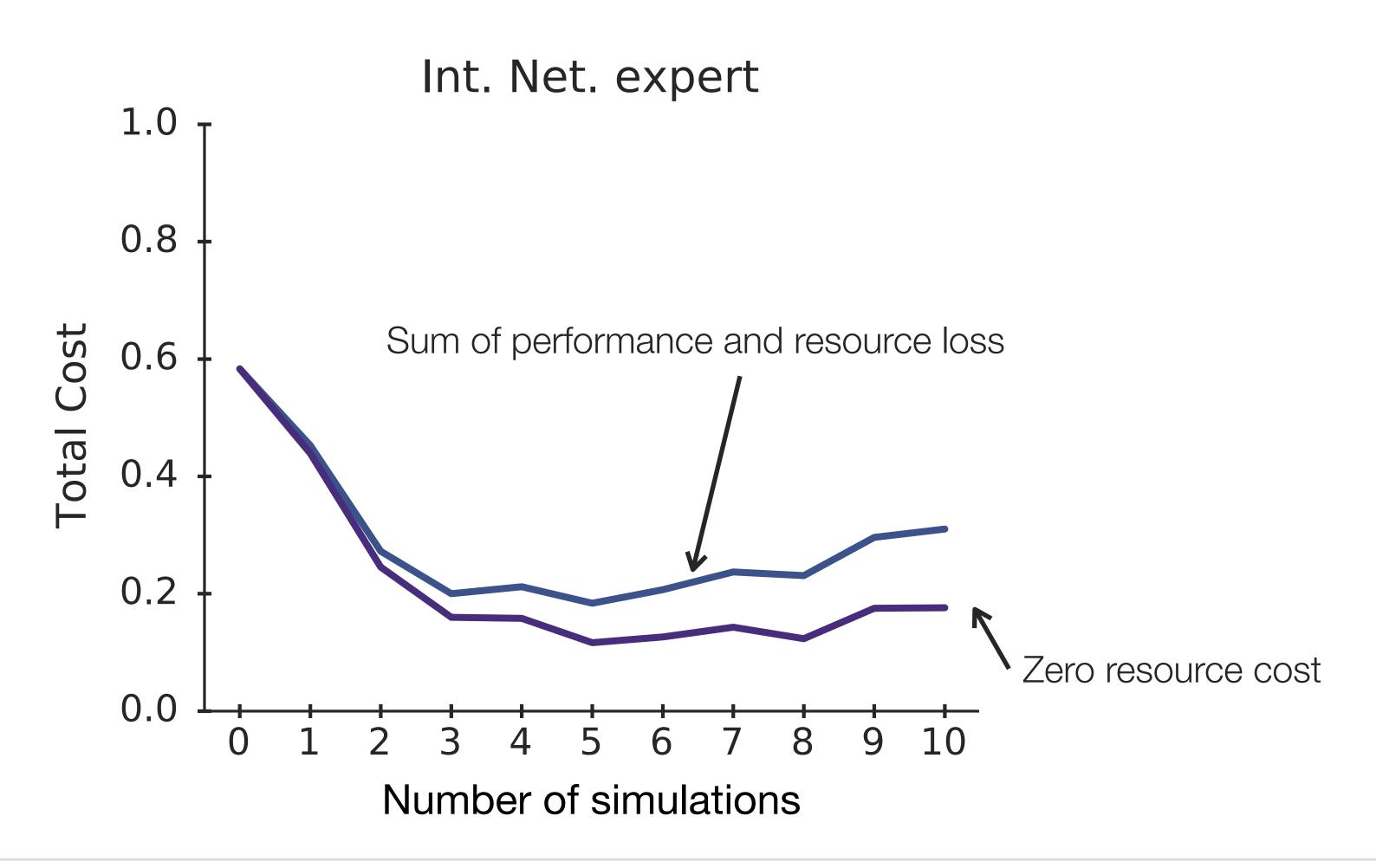




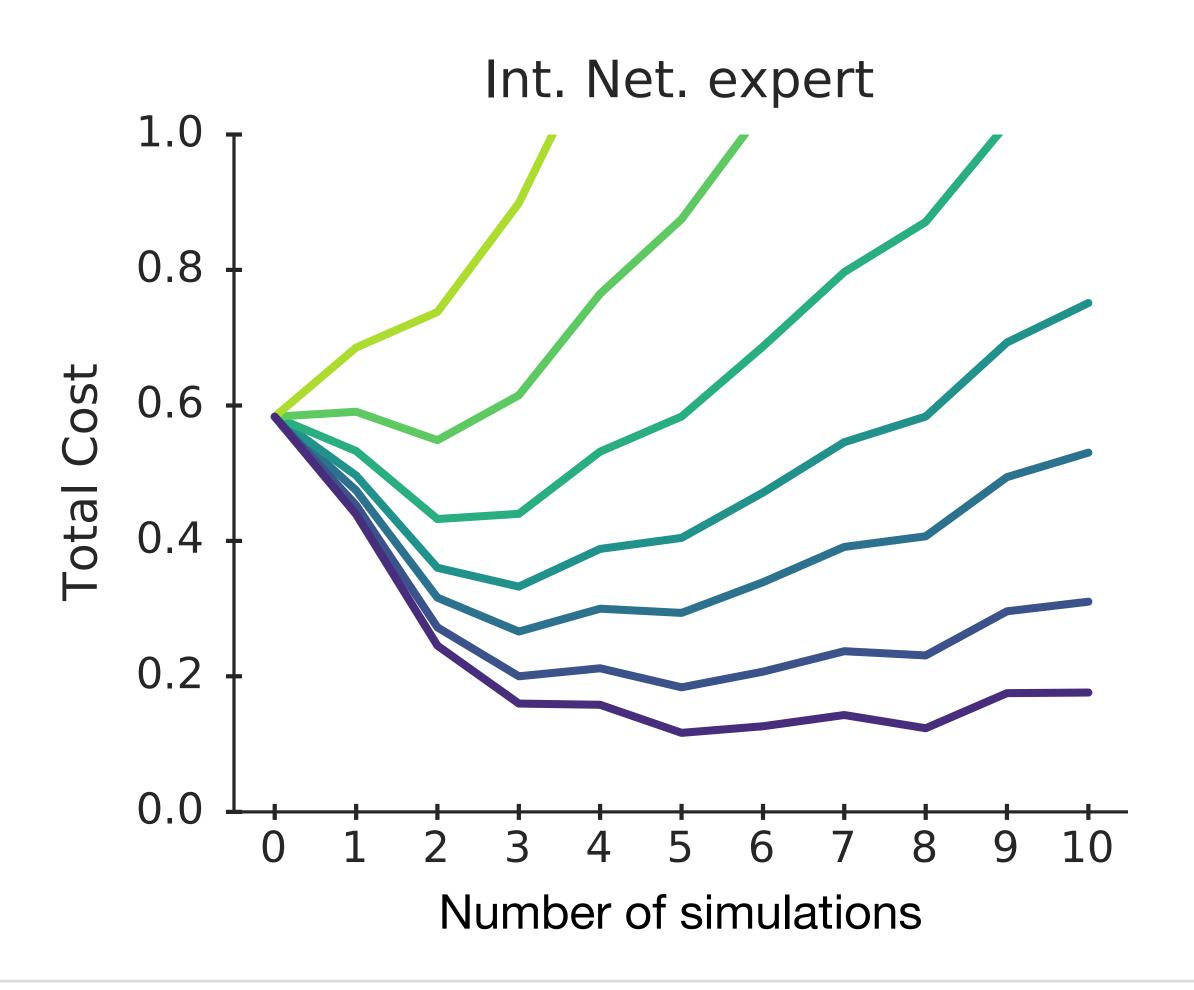




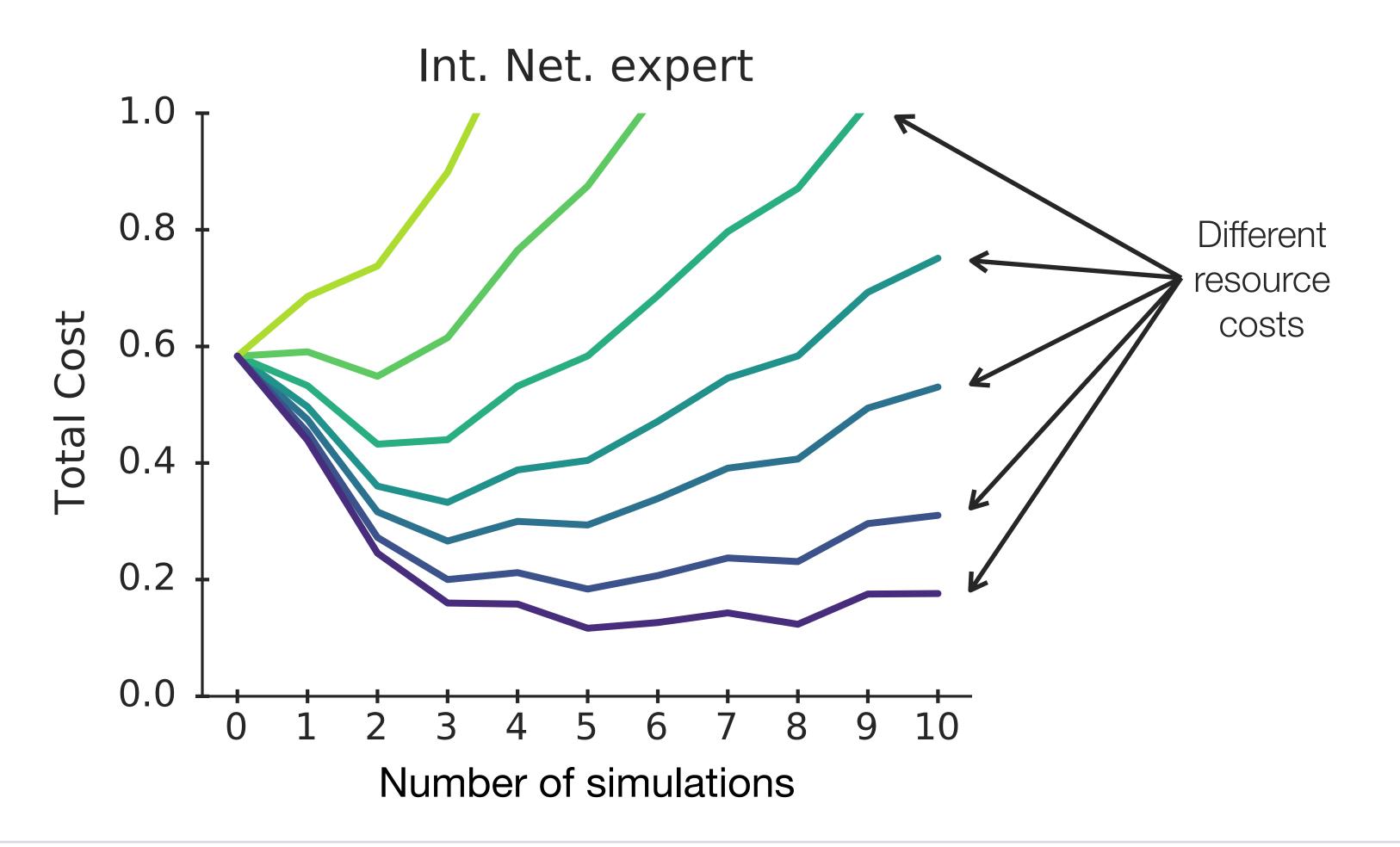




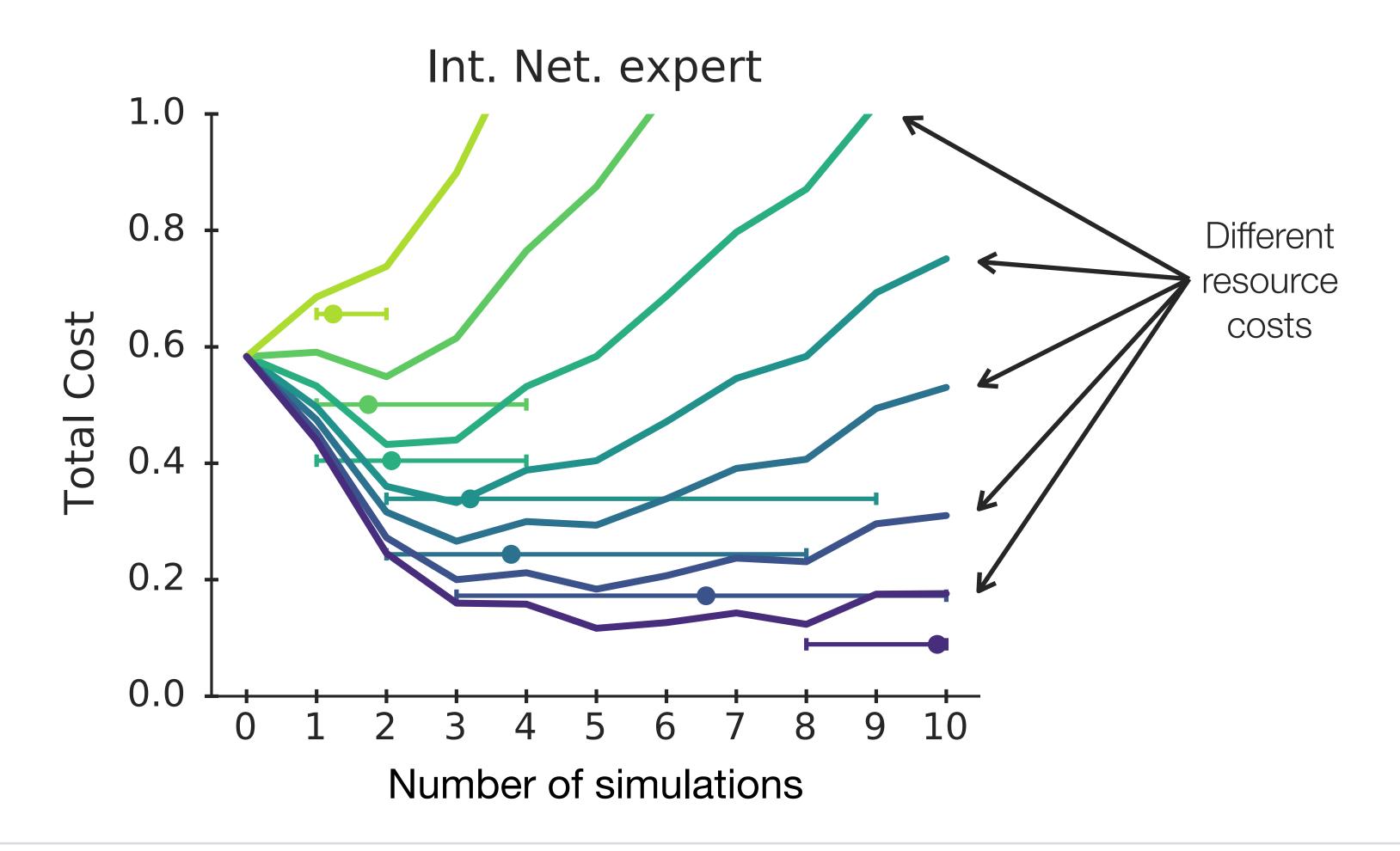














Multiple experts (MLP + IN)



Multiple experts (MLP + IN)

#### Total number of simulations

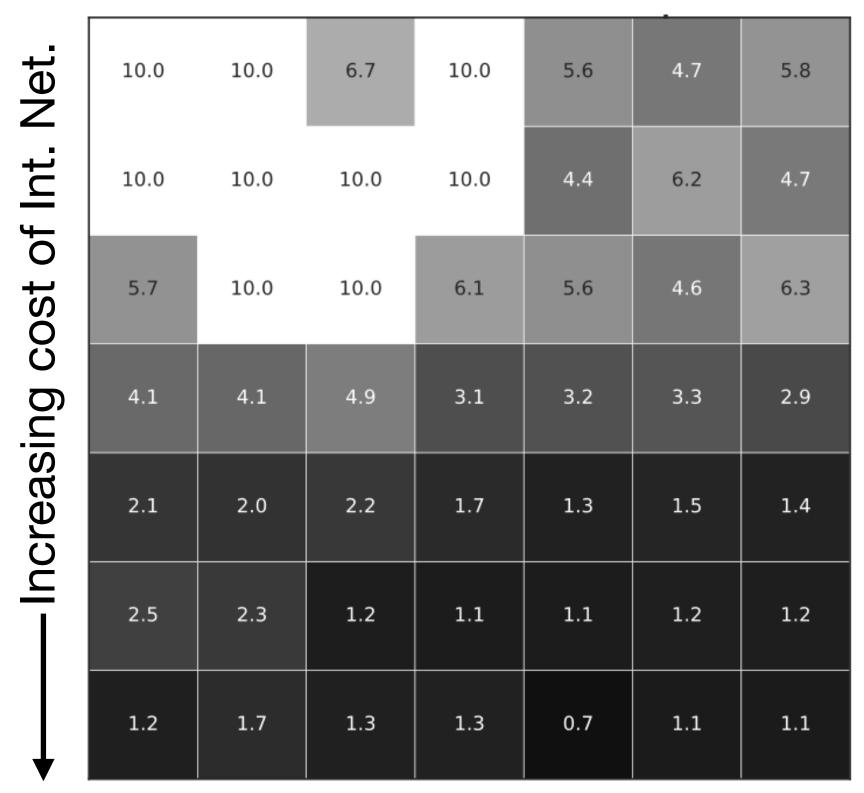
—Increasing cost of Int. Net.	10.0	10.0	6.7	10.0	5.6	4.7	5.8
	10.0	10.0	10.0	10.0	4.4	6.2	4.7
	5.7	10.0	10.0	6.1	5.6	4.6	6.3
	4.1	4.1	4.9	3.1	3.2	3.3	2.9
	2.1	2.0	2.2	1.7	1.3	1.5	1.4
	2.5	2.3	1.2	1.1	1.1	1.2	1.2
	1.2	1.7	1.3	1.3	0.7	1.1	1.1

Increasing cost of MLP ———



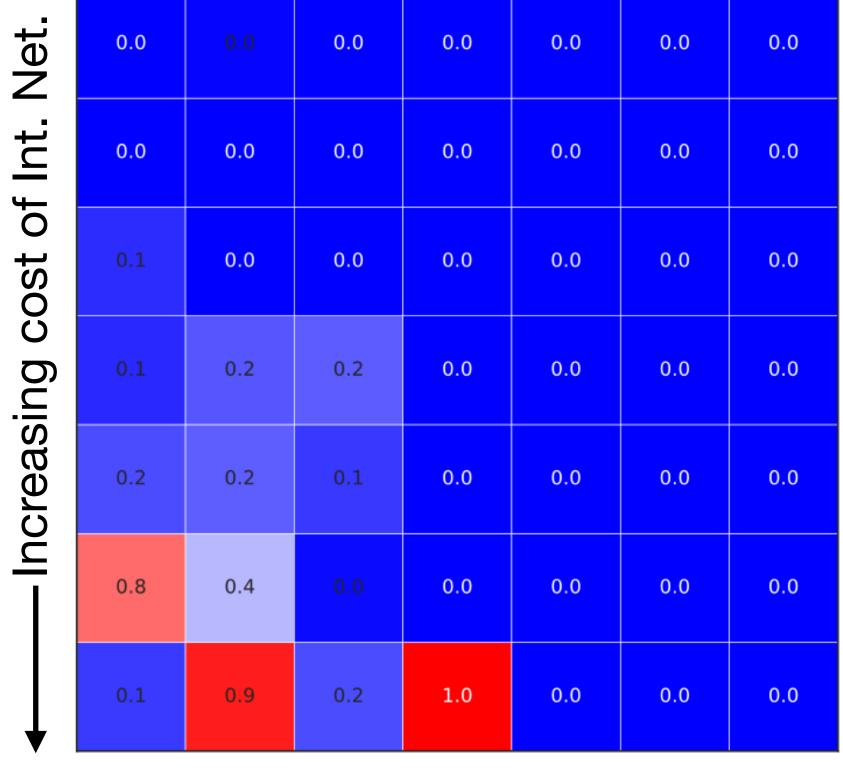
Multiple experts (MLP + IN)





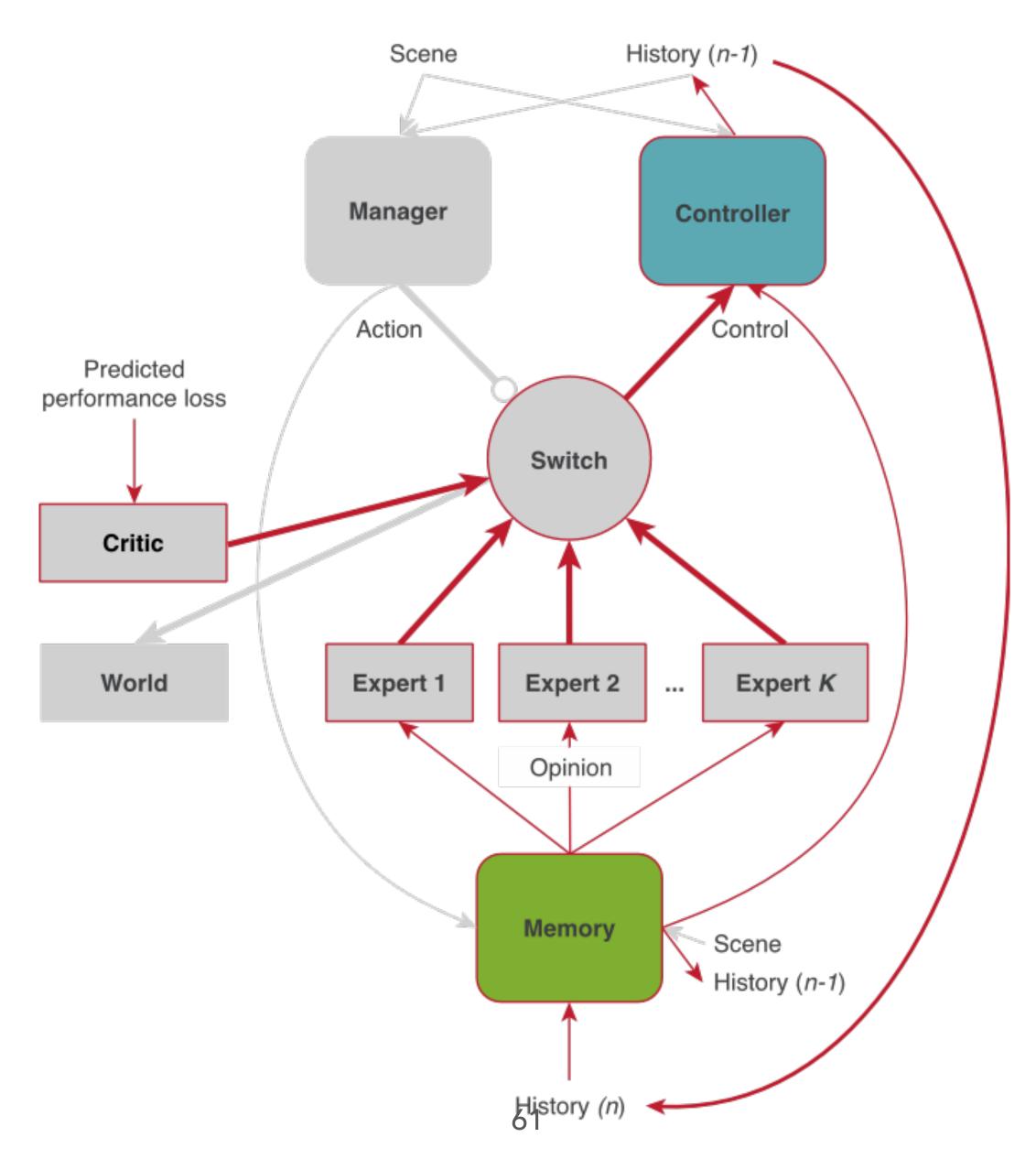
Increasing cost of MLP ———

Fraction of sims using MLP expert



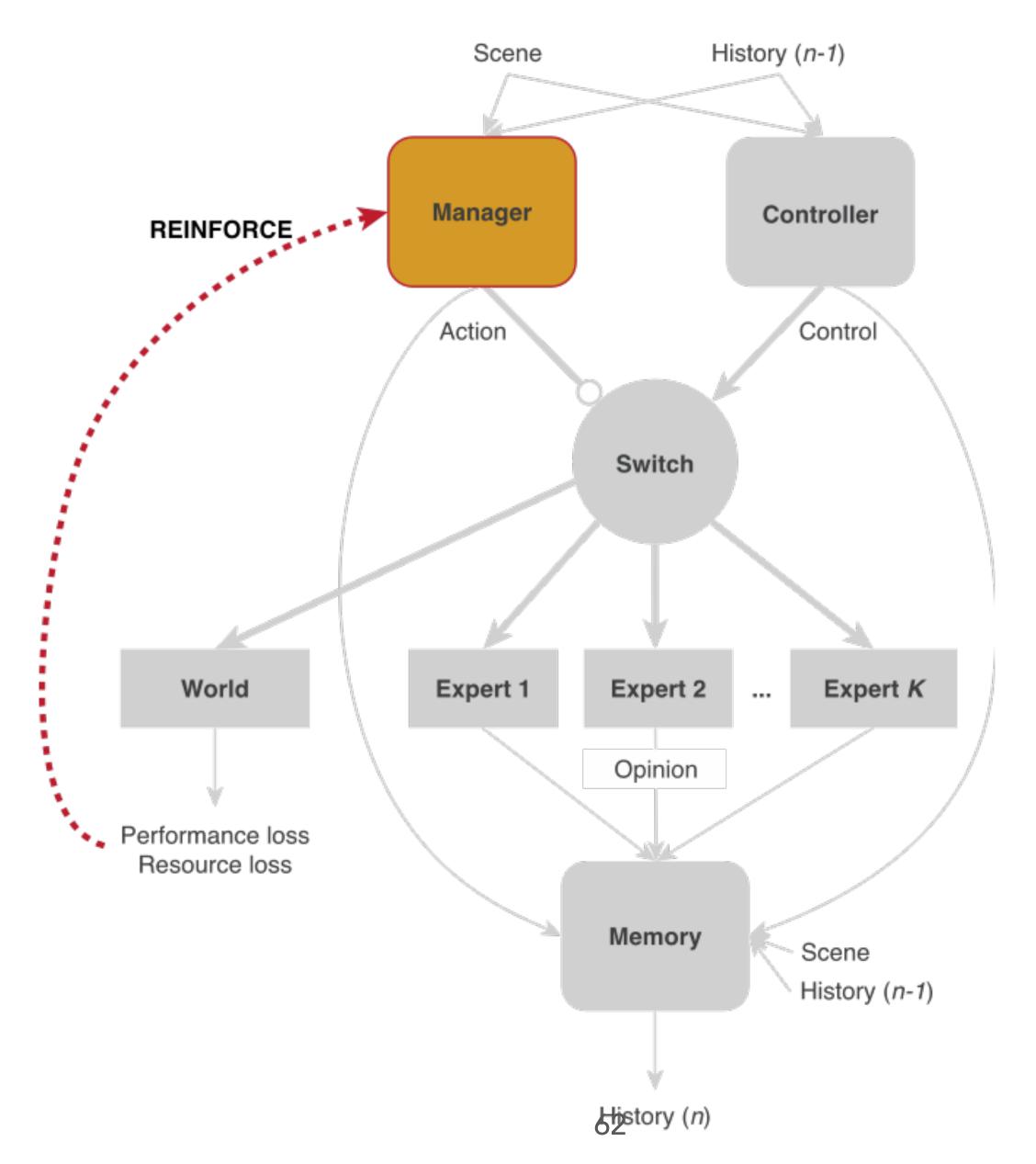
Increasing cost of MLP ———

## Training the Controller and Memory



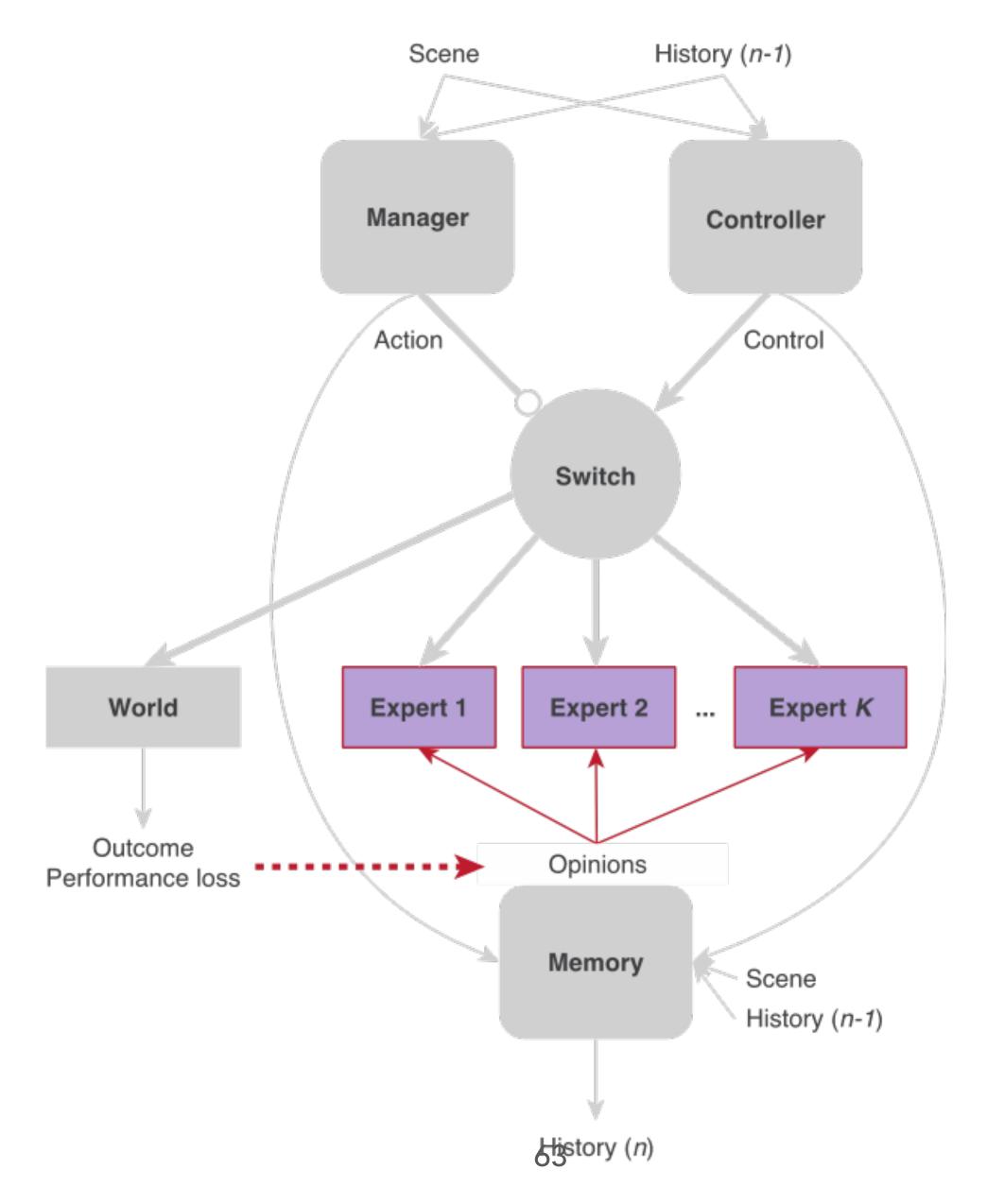


# Training the Manager





# Training the Experts





# Training the Critic

